NOTATION GUIDE FOR EDOS 5-72

(excerpted from Part V of "Alternative Tunings: Theory, Notation and Practice", on TallKite.com)

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Section 1 – Basic Definitions and the Scale Tree

The notations in this guide are all backwards compatible with conventional notation, to make it as easy as possible for the performer to read, and to make naming chords possible. For certain edos, the composer may prefer to compose in a notation that isn't backwards compatible, but communicate with performers in a notation that is. For example, 15-edo might use porcupine notation, generated not by 5ths but by 2nds, or 13-edo might use pentatonic or octotonic notation.

Edos are notated with conventional notation, plus **ups** and **downs** (^ and v). An up represents raising by exactly one **edostep** (one degree of the edo). Ups and downs are used in both absolute notation (^C, vB♭) and relative (^M3, v5). Interval arithmetic is mostly unchanged, with ups and downs adding up and canceling each other out as expected:

C to E = M3	C + M3 = E	M2 + M2 = M3
C to vE = vM3	C + vM3 = vE	M2 + vM2 = vM3
^C to ^E = M3	vC + ^M3 = E	vM2 + ^M2 = M3
^C to vE = vvM3	vC + vM3 = vvE	vM2 + vM2 = vvM3

Notes and intervals are spoken as "up C" or "downmajor 3rd". **Dup** and **dud** are short for double-up/double-down. Likewise there's **trup/trud**, **quup/quud** and **quip/quid**. A note or interval that is neither up nor down is **plain**.

There are 6 categories of edos:

superflat edos (9, 11, 13b, 16, 18b & 23) have a fifth narrower than four-sevenths of an octave = 4\7 = 686¢
perfect edos (7, 14, 21, 28 & 35) have a fifth of 4\7 = 686¢
diatonic edos (12, 17, 19, 22, 24, etc.) have a fifth that hits the "sweet spot" between 686¢ and 720¢
pentatonic edos (5, 10, 15, 20, 25 & 30) have a fifth of three-fifths of an octave = 3\5 = 720¢
supersharp edos (8, 13 & 18) have a fifth wider than 3\5 = 720¢
trivial edos (2, 3, 4 and 6) are notated as subsets of 12-edo

Supersharp edos have a descending minor 2nd, so conventional notation doesn't work. 8-edo is notated as a subset of 24-edo. 13-edo and 18-edo are notated using their second-best 5th, which converts them to the superflat edos 13b and 18b. Superflat edos have major intervals that are narrower than minor intervals. The sharp sign lowers the pitch and the flat sign raises it. However, ups still raise the pitch.

An interval's **keyspan** or **fretspan** is the number of edosteps the interval spans. The 12-edo keyspan of 6/5 is 3, and the 19-edo keyspan is 5.

Edos can be further categorized by the sharp symbol's keyspan, defined as 7 times the 5th's keyspan minus 4 times the octave's keyspan. 12-edo and 19-edo are **sharp-1**, 17-edo and 24-edo are sharp-2, and 16-edo is **flat-1**. All perfect edos are sharp-0. Sharp-1 and flat-1 edos don't need ups and downs.

The scale tree in the next picture shows all the edos ordered top to bottom, arranged left to right by the size of the 5th. The light gray lines are sharpness lines, connecting edos of similar sharpness. They spread out like ripples from the heptatonic kite, shown here in light blue.

Figure 1.1 – The Scale Tree from 650¢ to 750¢ and from 5-edo to 53-edo

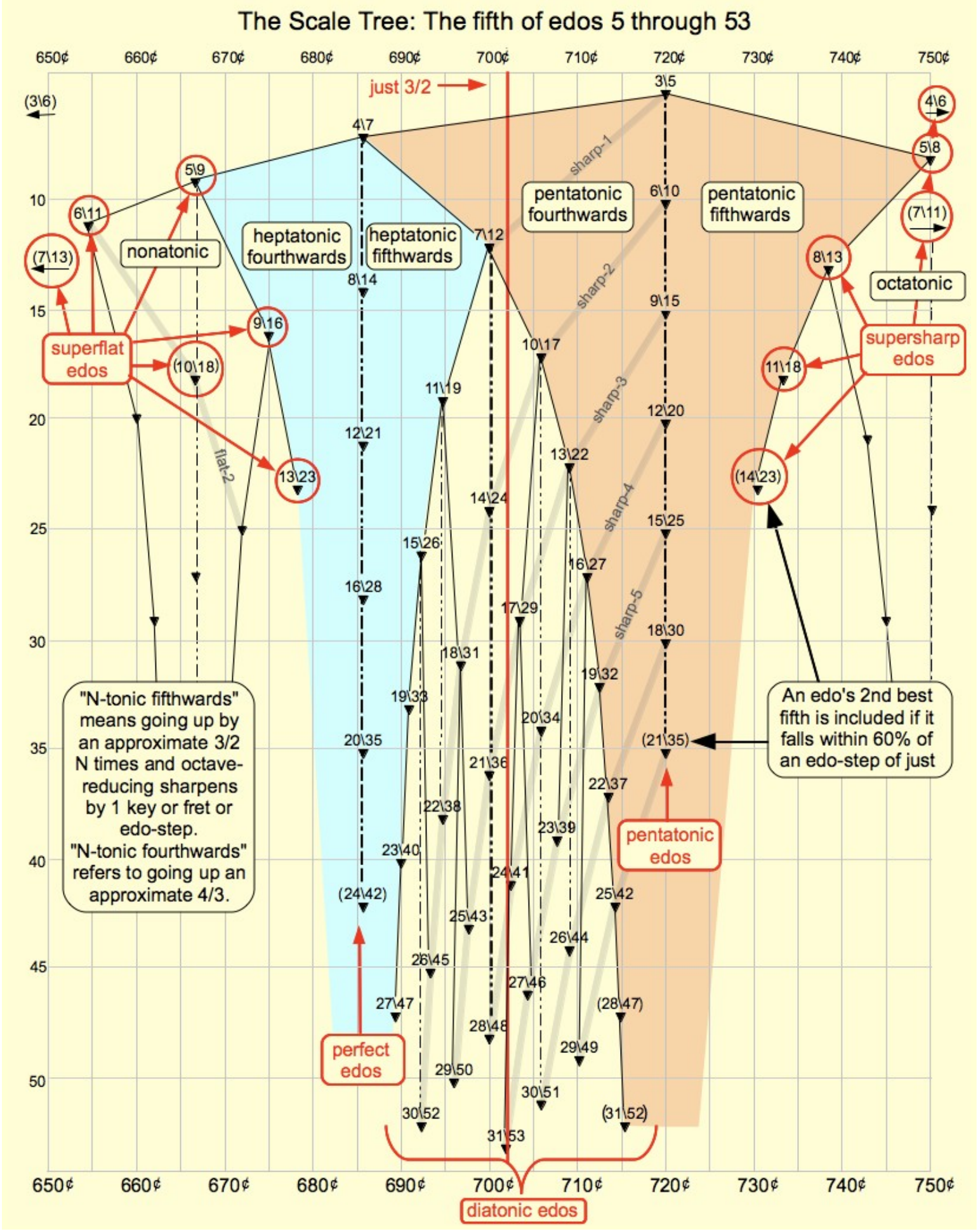
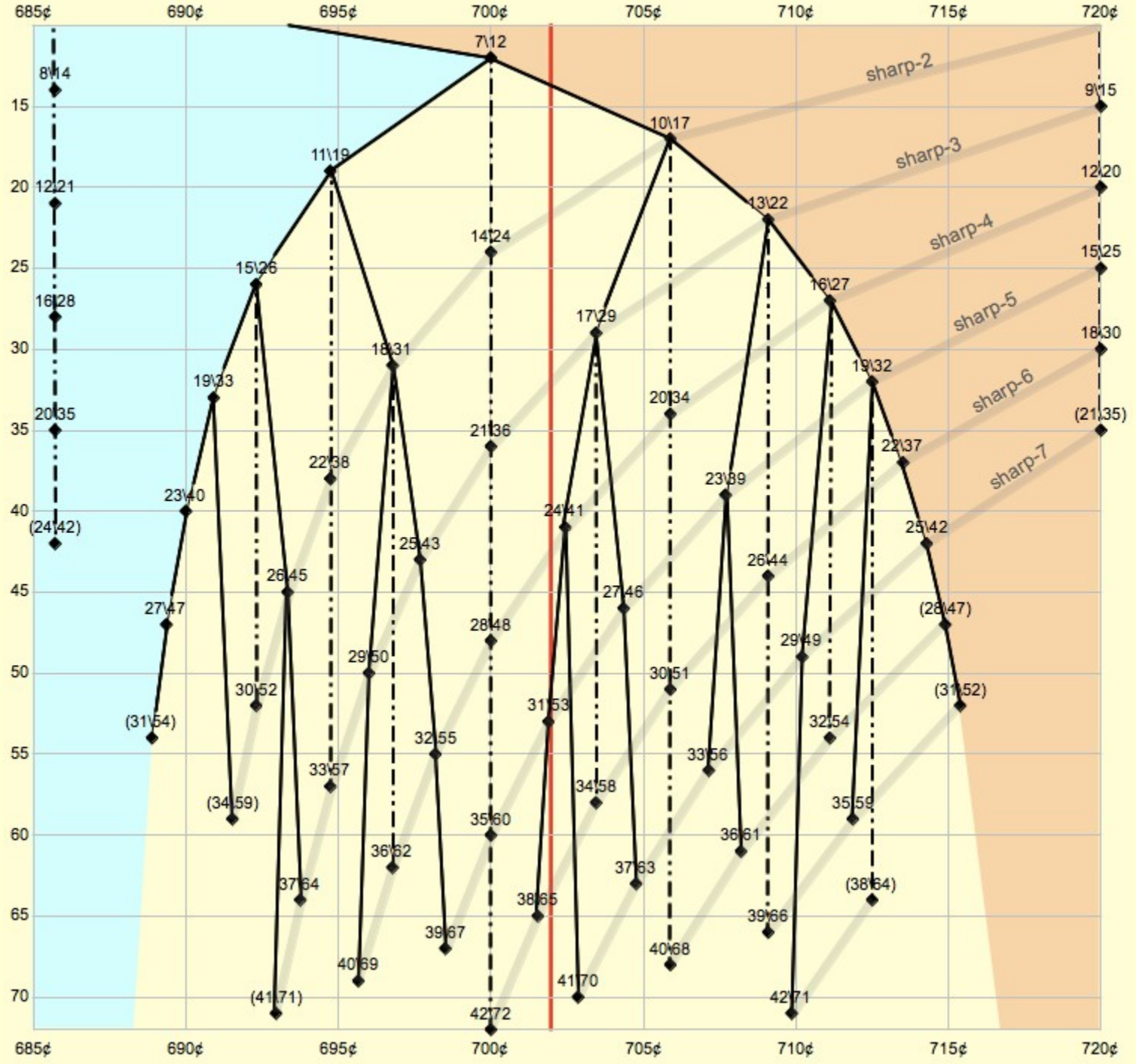


Figure 1.2 – The scale tree for edos 12-72, excluding supersharp and superflat ones

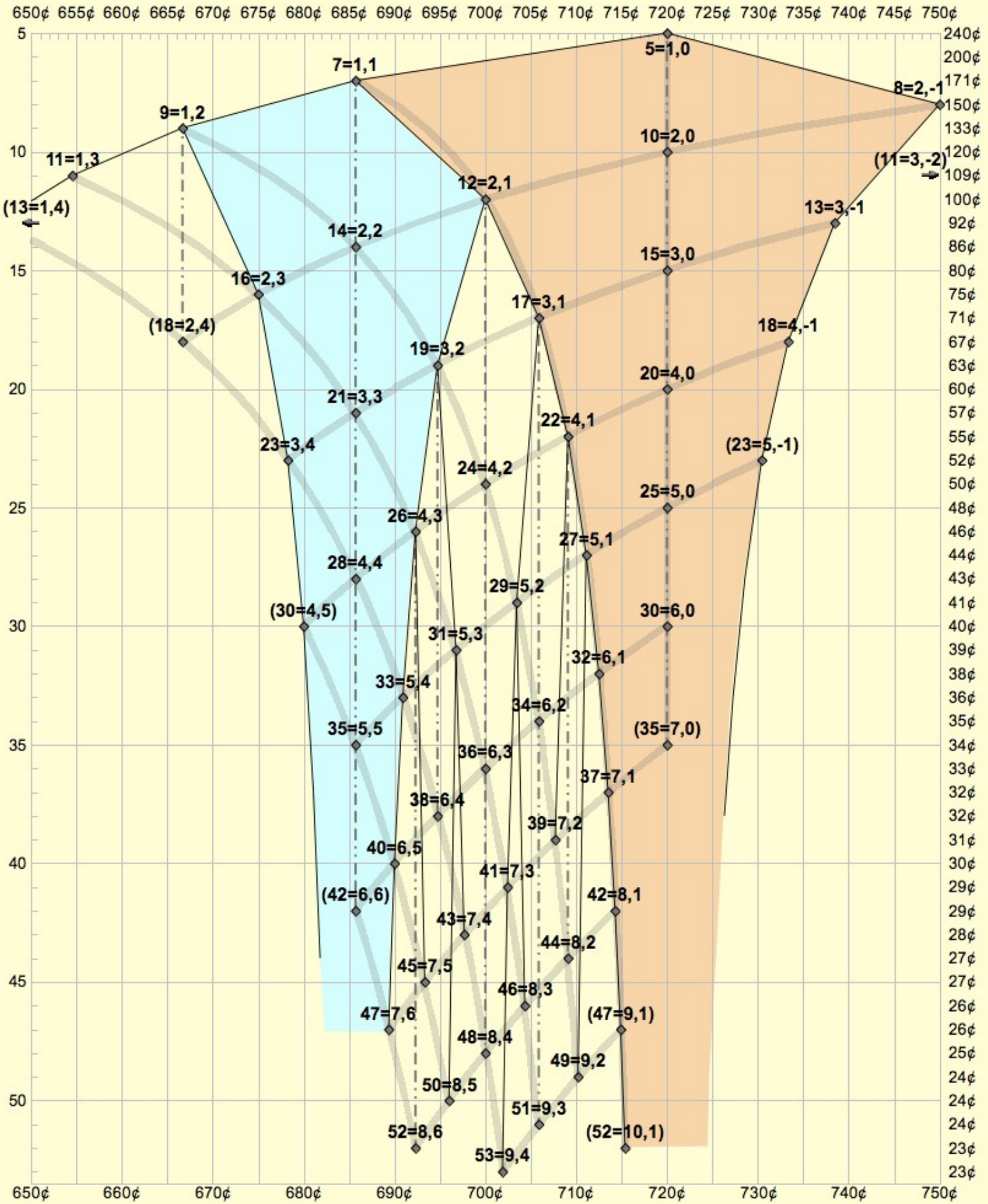


The next figure shows the size of the major and minor 2nds in edosteps. The edo's sharpness is simply the difference between these two numbers. The ratio of the two sizes depends directly on the size of the fifth, and increases steadily from right to left.

superflat edos have m2 > M2
perfect edos have m2 = M2
diatonic edos have m2 < M2
pentatonic edos have m2 = 0
supersharp edos have m2 < 0

Gray ripple lines connect edos with the same step sizes. The size in cents of each edo's edostep is shown on the right-hand side.

Figure 1.3 – The scale tree, showing the size of the major and minor 2nds in edosteps



Section 2 – The Notation Guide

Major, minor, perfect, etc. are known as qualities. Adjacent notes in an edo follow two **quality sequences**. For 12-edo, it's dim–minor–major–aug and dim–perfect–aug. Edos that use ups and downs have additional qualities like upmajor and dudaug. **Mid**, written ~, means exactly midway between major and minor, hence neutral. A mid 4th is midway between perfect and augmented, hence halfway augmented, and a mid 5th is halfway diminished. In sharp-2 edos, upminor equals downmajor, and mid replaces both terms. Some edos have qualities like upmid and downmid.

The next table shows the quality sequences for edos 5-72. **Dub** is short for double, and **trip** for triple. Diminished intervals can be deduced from augmented ones by symmetry. Likewise mid-5ths can be deduced from mid-4ths. Sharp-7 and higher edos are rarely used. Upmid and downmid are two edosteps apart in sharp-6 edos like 72, but only one edostep apart in sharp-5 edos like 53.

Table 2.1 – Quality sequences for edos 5-72, excluding 6-edo and 8-edo (**bold** = superflat edos)

category	edos	imperfect and perfect quality sequences (dim is symmetrical to aug)
sharp-0 (perfect)	7, 14, 21, 28, 35	(no imperfect intervals) perfect, up, dup, trup... = P, ^, ^^, ^3...
sharp-1, **flat-1**	5, **9**, 12, **16**, 19, **23**, 26, 33, 40, 47	minor, major, aug, dub-aug, trip-aug... = m, M, A, AA, A3... perfect, aug, dub-aug, trip-aug... = P, A, AA, A3...
sharp-2, **flat-2**	10, **11**, 17, **18b**, 24, 31, 38, 45, 52	minor, mid, major, upmajor, aug, up-aug, dub-aug... = m, ~, M, ^M, A, ^A, AA... perfect, up, aug, up-aug, dub-aug... = P, ^, A, ^A, AA… (up-4th is also called a mid-4th, down-5th is also called a mid-5th)
sharp-3, **flat-3**	**13b**, 15, 22, 29, 36, 43, 50, 57, 64	minor, upminor, downmajor, major, upmajor, downaug, aug... = m, ^m, vM, M, ^M, vA, A perfect, up, downaug, aug, up-aug, down-dub-aug, dub-aug... = P, ^, vA, A, ^A, vAA, AA...
sharp-4	20, 27, 34, 41, 48, 55, 62, 69	m, ^m, ~ (mid), vM, M, ^M, ^^M (dupmajor), vA, A... P, ^, ^^ (but ^^4 is ~4 and vv5 is ~5), vA, A, ^A, ^^A, vAA, AA...
sharp-5	25, 32, 39, 46, 53, 60, 67	m, ^m, v~ (downmid), ^~ (upmid), vM, M, ^M, ^^M, vvA, vA, A... P, ^, ^^ (but v~4), vvA (^~4), vA, A, ^A, ^^A, vvAA (dud dub-aug), vAA, AA...
sharp-6	30, 37, 44, 51, 58, 65, 72	m, ^m, v~, ~, ^~, vM, M, ^M, ^^M, ^3M (trupmajor), vvA, vA, A... P, ^, ^^ (v~4), ^3 (trup) (~4), vvA (^~4), vA, A, ^A, ^^A, ^3A, vvAA, vAA, AA...
sharp-7 (rare)	42, 49, 56, 63, 70	m, ^m, ^^m, v~, ^~, vvM, vM, M, ^M, ^^M, ^3M, v3A, vvA, vA, A... P, ^, ^^, ^3 (v~4), v3A (^~4), vvA, vA, A, ^A, ^^A, ^3A, v3AA, vvAA, vAA, AA...
sharp-8 (rare)	54, 61, 68	m, ^m, ^^m, v~, ~, ^~, vvM, vM, M, ^M, ^^M, ^3M, ^4M, v3A, vvA, vA, A... P, ^, ^^, ^3 (v~4), ^4 (~4), v3A (^~4), vvA, vA, A, ^A, ^^A, ^3A, ^4A, v3AA, vvAA, vAA, AA
sharp-9 (rare)	59, 66	m, ^m, ^^m, vv~, v~, ^~, ^^~, vvM, vM, M, ^M, ^^M, ^3M, ^4M, v4A, v3A, vvA, vA, A... P, ^, ^^, ^3, ^4, v4A, v3A, vvA, vA, A, ^A, ^^A, ^3A, ^4A, v4AA, v3AA, vvAA, vAA, AA...
sharp-10 (rare)	71	m, ^m, ^^m, vv~, v~, ~, ^~, ^^~, vvM, vM, M, ^M, ^^M, ^3M, ^4M, ^5M, v4A, v3A, vvA, vA P, ^, ^^, ^3, ^4, ^5, v4A, v3A, vvA, vA, A, ^A, ^^A, ^3A, ^4A, ^5A, v4AA, v3AA, vvAA...

The next table is a notation guide for edos 5 through 72. Every note from D to F is represented in at least two ways, and often three ways. The full octave can be found by extrapolating the D–E major 2nd and the E–F minor 2nd. Triple sharps/flats and triple ups/downs use exponents, both in text and in staff notation:

^^D = dup D = ^^M2, if in the key of C = dupmajor 2nd
^^D^{X} = dup D dub-sharp = ^^AA2 = dup dub-aug 2nd
v^{3}D♯ = trud D sharp = v^{3}A2 = trud aug 2nd
vD$^{♯3}$ = down D trip-sharp = vA^{3}2 = down trip-aug 2nd

Table 2.2 – Notation Guide for Edos 5-72 Using Ups and Downs, Showing White and Black Keys

5-edo	pentatonic sharp-1	**D** E♭ F♭	D♯ **E** **F**					
6-edo	trivial (subset of 12-edo)	**D** E♭♭	D𝄪 **E** F♭	E𝄪 F♯				
7-edo	perfect sharp-0	**D**	**E**	**F**				
8-edo	supersharp (subset of 24-edo)	**D** E♭♭	^D♯ vE vF♭	E♯ **F**				
9-edo	superflat flat-1	**D** E♯	D♭ **E** F𝄪	D♭♭ E♭ F♯	E♭♭ **F**			
10-edo	pentatonic sharp-2	**D** E♭ F♭	^D vE vF	D♯ **E** **F**				
11-edo	superflat flat-2	**D** vE	^D **E** vF♯	D♭ ^E F♯	^D♭ E♭ vF	^E♭ **F**		
12-edo	diatonic sharp-1	**D** E♭♭	D♯ E♭ F♭♭	D𝄪 **E** F♭	E♯ **F**			
13b-edo (5th = 7\13)	superflat flat-3	**D** vE	^D **E** vF♯	vD♭ ^E F♯	D♭ vE♭ ^F♯	^D♭ E♭ vF	^E♭ **F**	
14-edo	perfect sharp-0	**D** vvE	^D vE v3F	^^D **E** vvF	^3D ^E vF	^^E **F**		
15-edo	pentatonic sharp-3	**D** E♭ F♭	^D ^E♭ ^F♭	vD♯ vE vF	D♯ **E** **F**			
16-edo	superflat flat-1	**D** E𝄪	D♭ E♯	D♭♭ **E** F♯3	D♭3 E♭ F𝄪	E♭♭ F♯	E♭3 **F**	
17-edo	diatonic sharp-2	**D** vE♭ F♭♭	^D E♭ vF♭	D♯ vE F♭	^D♯ **E** vF	D𝄪 ^E **F**		
18b-edo (5th = 10\18)	superflat flat-2	**D** E♯	^D vE vF𝄪	D♭ **E** F𝄪	^D♭ ^E vF♯	D♭♭ E♭ F♯	^D♭♭ ^E♭ vF	E♭♭ **F**

19-edo	diatonic sharp-1	**D** E♭3	D♯ E♭♭	D𝄪 E♭ F♭3	D♯3 **E** F♭♭	E♯ F♭	E𝄪 **F**				
20-edo	pentatonic sharp-4	**D** E♭ F♭	^D ^E♭ ^F♭	^^D vvE vvF	vD♯ vE vF	D♯ **E** **F**					
21-edo	perfect sharp-0	**D** v3E	^D vvE	^^D vE v4F	^3D **E** v3F	^4D ^E vvF	^^E vF	^3E **F**			
22-edo	diatonic sharp-3	**D** vE♭ ^F♭♭	^D E♭ vF♭	vD♯ ^E♭ F♭	D♯ vE ^F♭	^D♯ **E** vF	vD𝄪 ^E **F**				
23-edo	superflat flat-1	**D** E♯3	D♭ E𝄪	D♭♭ E♯	D♭3 **E** F♯4	D♭4 E♭ F♯3	E♭♭ F𝄪	E♭3 F♯	E♭4 **F**		
24-edo	diatonic sharp-2	**D** E♭♭	^D vE♭	D♯ E♭ F♭♭	^D♯ vE vF♭	D𝄪 **E** F♭	^E vF	E♯ **F**			
25-edo	pentatonic sharp-5	**D** E♭ F♭	^D ^E♭ ^F♭	^^D ^^E♭ ^^F♭	vvD♯ vvE vvF	vD♯ vE vF	D♯ **E** **F**				
26-edo	diatonic sharp-1	**D** E♭4	D♯ E♭3	D𝄪 E♭♭	D♯3 E♭ F♭4	D♯4 **E** F♭3	E♯ F♭♭	E𝄪 F♭	E♯3 **F**		
27-edo	diatonic sharp-4	**D** vE♭ vvF♭	^D E♭ vF♭	^^D ^E♭ F♭	vD♯ vvE ^F♭	D♯ vE vvF	^D♯ **E** vF	^^D♯ ^E **F**			
28-edo	perfect sharp-0	**D** v4E	^D v3E	^^D vvE	^3D vE v5F	^4D **E** v4F	^5D ^E v3F	^^E vvF	^3E vF	^4E **F**	
29-edo	diatonic sharp-3	**D** ^E♭♭	^D vE♭ F♭♭	vD♯ E♭ ^F♭♭	D♯ ^E♭ vF♭	^D♯ vE F♭	vD𝄪 **E** ^F♭	D𝄪 ^E vF	vE♯ **F**		
30-edo	pentatonic sharp-6	**D** E♭ F♭	^D ^E♭ ^F♭	^^D ^^E♭ ^^F♭	^3D v3E v3F	vvD♯ vvE vvF	vD♯ vE vF	D♯ **E** **F**			
31-edo	diatonic sharp-2	**D** vE♭♭	^D E♭♭	D♯ vE♭	^D♯ E♭ vF♭♭	D𝄪 vE F♭♭	^D𝄪 **E** vF♭	^E F♭	E♯ vF	^E♯ **F**	
32-edo	diatonic sharp-5	**D** vE♭	^D E♭ vF♭	^^D ^E♭ F♭	vvD♯ ^^E♭ ^F♭	vD♯ vvE ^^F♭	D♯ vE vvF	^D♯ **E** vF	^E **F**		
33-edo	diatonic sharp-1	**D** E♭5	D♯ E♭4	D𝄪 E♭3	D♯3 E♭♭ F♭6	D♯4 E♭ F♭5	D♯5 **E** F♭4	D♯6 E♯ F♭3	E𝄪 F♭♭	E♯3 F♭	E♯4 **F**

	0	1	2	3	4	5	6	7	8	9	10			
34-edo diatonic sharp-4	**D** vvE♭ F♭♭	^D vE♭ ^F♭♭	^^D E♭ vvF♭	vD♯ ^E♭ vF♭	D♯ vvE F♭	^D♯ vE ^F♭	^^D♯ **E** vvF	vD𝄪 ^E vF	D𝄪 ^^E **F**					
35-edo perfect sharp-0	**D** v5E	^D v4E	^^D v3E	^3D vvE vF7	^4D vE vF6	^5D **E** v5F	^D6 ^E v4F	^D7 ^^E v3F	^3E vvF	^4E vF	^5E **F**			
36-edo sharp-3	**D** E♭♭	^D ^E♭♭	vD♯ vE♭	D♯ E♭ F♭♭	^D♯ ^E♭ ^F♭♭	vD𝄪 vE vF♭	D𝄪 **E** F♭	^E ^F♭	vE♯ vF	E♯ **F**				
37-edo sharp-6	**D** vE♭ vvF♭	^D E♭ vF♭	^^D ^E♭ F♭	^3D ^^E♭ ^F♭	vvD♯ v3E ^^F♭	vD♯ vvE v3F	D♯ vE vvF	^D♯ **E** vF	^^D♯ ^E **F**					
38-edo sharp-2	**D** E♭3	^D vE♭♭	D♯ E♭♭ F♭4	^D♯ vE♭ vF♭3	D𝄪 E♭ F♭3	^D𝄪 vE vF♭♭	D♯3 **E** F♭♭	^D♯3 ^E vF♭	D♯4 E♯ F♭	^E♯ vF	E𝄪 **F**			
39-edo sharp-5	**D** vvE♭	^D vE♭	^^D E♭ vvF♭	vvD♯ ^E♭ vF♭	vD♯ ^^E♭ F♭	D♯ vvE ^F♭	^D♯ vE ^^F♭	^^D♯ **E** vvF	^E vF	^^E **F**				
40-edo sharp-1	**D** E♭6	D♯ E♭5	D𝄪 E♭4	D♯3 E♭3	D♯4 E♭♭ F♭7	D♯5 E♭ F♭6	D♯6 **E** F♭5	D♯7 E♯ F♭4	E𝄪 F♭3	E♯3 F♭♭	E♯4 F♭	E♯5 **F**		
41-edo sharp-4	**D** ^E♭♭	^D vvE♭	^^D vE♭ F♭♭	vD♯ E♭ ^F♭♭	D♯ ^E♭ vvF♭	^D♯ vvE vF♭	^^D♯ vE F♭	vD𝄪 **E** ^F♭	D𝄪 ^E vvF	^^E vF	vE♯ **F**			
42-edo sharp-7	**D** vE♭	^D E♭ vF♭	^^D ^E♭ F♭	^3D ^^E♭ ^F♭	v3D♯ ^3E♭ ^^F♭	vvD♯ v3E ^3F♭	vD♯ vvE v3F	D♯ vE vvF	^D♯ **E** vF	^E **F**				
43-edo sharp-3	**D** vE♭♭	^D E♭♭	vD♯ ^E♭♭	D♯ vE♭ ^F♭3	^D♯ E♭ vF♭♭	vD𝄪 ^E♭ F♭♭	D𝄪 vE ^F♭♭	^D𝄪 **E** vF♭	vD♯3 ^E F♭	vE♯ ^F♭	E♯ vF	^E♯ **F**		
44-edo sharp-6	**D** vvE♭	^D vE♭	^^D E♭ vvF♭	^3D ^E♭ vF♭	vvD♯ ^^E♭ F♭	vD♯ v3E ^F♭	D♯ vvE ^^F♭	^D♯ vE v3F	^^D♯ **E** vvF	^E vF	^^E **F**			
45-edo sharp-2	**D** vE♭3	^D E♭3	D♯ vE♭♭	^D♯ E♭♭ vF♭4	D𝄪 vE♭ F♭4	^D𝄪 E♭ vF♭3	D♯3 vE F♭3	^D♯3 **E** vF♭♭	D♯4 ^E F♭♭	^D♯4 E♯ vF♭	^E♯ F♭	E𝄪 vF	^E𝄪 **F**	
46-edo sharp-5	**D** ^^E♭♭	^D vvE♭	^^D vE♭ ^F♭♭	vvD♯ E♭ ^^F♭♭	vD♯ ^E♭ vvF♭	D♯ ^^E♭ vF♭	^D♯ vvE F♭	^^D♯ vE ^F♭	vvD𝄪 **E** ^^F♭	vD𝄪 ^E vvF	^^E vF	vvE♯ **F**		
47-edo sharp-1	**D** E♭7	D♯ E♭6	D𝄪 E♭5	D♯3 E♭4	D♯4 E♭3 F♭9	D♯5 E♭♭ F♭8	D♯6 E♭ F♭7	D♯7 **E** F♭6	D♯8 E♯ F♭5	D♯9 E𝄪 F♭4	E♯3 F♭3	E♯4 F♭♭	E♯5 F♭	E♯6 **F**
	0	1	2	3	4	5	6	7	8	9	10	11	12	13

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
48-edo ♯ = 4	**D** E𝄫	^D ^E𝄫	^^D vvE𝄫	vD♯ vE𝄫 vF𝄫	D♯ E𝄫 F𝄫	^D♯ ^E𝄫 ^F𝄫	^^D♯ vvE vvF𝄫	vD𝄪 vE vF𝄫	D𝄪 **E** F𝄫	^D𝄪 ^E ^F𝄫	^^E vvF	vE♯ vF	E♯ **F**			
49-edo ♯ = 7	**D** vvE𝄫	^D vE𝄫	^^D E𝄫 vvF𝄫	^3D ^E𝄫 vF𝄫	v3D♯ ^^E𝄫 F𝄫	vvD♯ ^3E𝄫 ^F𝄫	vD♯ v3E ^^F𝄫	D♯ vvE ^3F𝄫	^D♯ vE v3F	^^D♯ **E** vvF	^E vF	^^E **F**				
50-edo ♯ = 3	**D** ^E𝄫3	^D vE𝄫𝄫	vD♯ E𝄫𝄫	D♯ ^E𝄫𝄫	^D♯ vE𝄫 F𝄫3	vD𝄪 E𝄫 ^F𝄫3	D𝄪 ^E𝄫 vF𝄫𝄫	^D𝄪 vE F𝄫𝄫	vD♯3 **E** ^F𝄫𝄫	D♯3 ^E vF𝄫	vE♯ F𝄫	E♯ ^F𝄫	^E♯ vF	vE𝄪 **F**		
51-edo ♯ = 6	**D** v3E𝄫	^D vvE𝄫	^^D vE𝄫	^3D E𝄫 v3F𝄫	vvD♯ ^E𝄫 vvF𝄫	vD♯ ^^E𝄫 vF𝄫	D♯ v3E F𝄫	^D♯ vvE ^F𝄫	^^D♯ vE ^^F𝄫	^3D♯ **E** v3F	^E vvF	^^E vF	^3E **F**			
52-edo ♯ = 2	**D** E𝄫4	^D vE𝄫3	D♯ E𝄫3	^D♯ vE𝄫𝄫	D𝄪 E𝄫𝄫 F𝄫5	^D𝄪 vE𝄫 vF𝄫4	D♯3 E𝄫 F𝄫4	^D♯3 vE vF𝄫3	D♯4 **E** F𝄫3	^D♯4 ^E vF𝄫𝄫	D♯5 E♯ F𝄫𝄫	^E♯ vF𝄫	E𝄪 F𝄫	^E𝄪 vF	E♯3 **F**	
53-edo ♯ = 5	**D** ^E𝄫𝄫	^D ^^E𝄫𝄫	^^D vvE𝄫	vvD♯ vE𝄫 F𝄫𝄫	vD♯ E𝄫 ^F𝄫𝄫	D♯ ^E𝄫 ^^F𝄫𝄫	^D♯ ^^E𝄫 vvF𝄫	^^D♯ vvE vF𝄫	vvD𝄪 vE F𝄫	vD𝄪 **E** ^F𝄫	D𝄪 ^E ^^F𝄫	^^E vvF	vvE♯ vF	vE♯ **F**		
54-edo ♯ = 8	**D** vvE𝄫	^D vE𝄫	^^D E𝄫 vvF𝄫	^3D ^E𝄫 vF𝄫	^4D ^^E𝄫 F𝄫	v3D♯ ^3E𝄫 ^F𝄫	vvD♯ v4E ^^F𝄫	vD♯ v3E ^3F𝄫	D♯ vvE v4F	^D♯ vE v3F	^^D♯ **E** vvF	^E vF	^^E **F**			
55-edo ♯ = 4	**D** vE𝄫𝄫	^D E𝄫𝄫	^^D ^E𝄫𝄫	vD♯ vvE𝄫	D♯ vE𝄫 vvF𝄫𝄫	^D♯ E𝄫 vF𝄫𝄫	^^D♯ ^E𝄫 F𝄫𝄫	vD𝄪 vvE ^F𝄫𝄫	D𝄪 vE vvF𝄫	^D𝄪 **E** vF𝄫	^^D𝄪 ^E F𝄫	^^E ^F𝄫	vE♯ vvF	E♯ vF	^E♯ **F**	
56-edo ♯ = 7	**D** v3E𝄫	^D vvE𝄫	^^D vE𝄫	^3D E𝄫 v3F𝄫	v3D♯ ^E𝄫 vvF𝄫	vvD♯ ^^E𝄫 vF𝄫	vD♯ ^3E𝄫 F𝄫	D♯ v3E ^F𝄫	^D♯ vvE ^^F𝄫	^^D♯ vE ^3F𝄫	^3D♯ **E** v3F	^E vvF	^^E vF	^3E **F**		
57-edo ♯ = 3	**D** E𝄫3	^D ^E𝄫3	vD♯ vE𝄫𝄫	D♯ E𝄫𝄫 F𝄫4	^D♯ ^E𝄫𝄫 ^F𝄫4	vD𝄪 vE𝄫 vF𝄫3	D𝄪 E𝄫 F𝄫3	^D𝄪 ^E𝄫 ^F𝄫3	vD♯3 vE vF𝄫𝄫	D♯3 **E** F𝄫𝄫	^D♯3 ^E ^F𝄫𝄫	vD♯4 vE♯ vF𝄫	D♯4 E♯ F𝄫	^E♯ ^F𝄫	vE𝄪 vF	E𝄪 **F**
58-edo ♯ = 6	**D** ^^E𝄫𝄫	^D v3E𝄫	^^D vvE𝄫	^3D vE𝄫 ^F𝄫𝄫	vvD♯ E𝄫 ^^F𝄫𝄫	vD♯ ^E𝄫 v3F𝄫	D♯ ^^E𝄫 vvF𝄫	^D♯ v3E vF𝄫	^^D♯ vvE F𝄫	^3D♯ vE ^F𝄫	vvD𝄪 **E** ^^F𝄫	vD𝄪 ^E v3F	^^E vvF	^3E vF	vvE♯ **F**	
59-edo ♯ = 9	**D** vvE𝄫	^D vE𝄫	^^D E𝄫 vvF𝄫	^3D ^E𝄫 vF𝄫	^4D ^^E𝄫 F𝄫	v4D♯ ^3E𝄫 ^F𝄫	v3D♯ ^4E𝄫 ^^F𝄫	vvD♯ v4E ^3F𝄫	vD♯ v3E ^4F𝄫	D♯ vvE v4F	^D♯ vE v3F	^^D♯ **E** vvF	^E vF	^^E **F**		
60-edo ♯ = 5	**D** E𝄫𝄫	^D ^E𝄫𝄫	^^D ^^E𝄫𝄫	vvD♯ vE𝄫	vD♯ vE𝄫 vF𝄫𝄫	D♯ E𝄫 F𝄫𝄫	^D♯ ^E𝄫 ^F𝄫𝄫	^^D♯ ^^E𝄫 ^^F𝄫𝄫	vvD𝄪 vvE vvF𝄫	vD𝄪 vE vF𝄫	D𝄪 **E** F𝄫	^D𝄪 ^E ^F𝄫	^^E ^^F𝄫	vvE♯ vvF	vE♯ vF	E♯ **F**
61-edo ♯ = 8	**D** v3E𝄫	^D vvE𝄫	^^D vE𝄫	^3D E𝄫 v3F𝄫	^4D ^E𝄫 vvF𝄫	v3D♯ ^^E𝄫 vF𝄫	vvD♯ ^3E𝄫 F𝄫	vD♯ v4E ^F𝄫	D♯ v3E ^^F𝄫	^D♯ vvE ^3F𝄫	^^D♯ vE v4F	^3D♯ **E** v3F	^E vvF	^^E vF	^3E **F**	
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
62 ♯ = 4	**D** vvE♭♭	^D vE♭♭	^^D E♭♭	vD♯ ^E♭♭	D♯ vvE♭ F♭3	^D♯ vE♭ ^F♭3	^^D♯ E♭ vvF♭♭	vD𝗫 ^E♭ vF♭♭	D𝗫 vvE F♭♭	^D𝗫 vE ^F♭♭	^^D𝗫 **E** vvF♭	vD♯3 ^E vF♭	D♯3 ^^E F♭	vE♯ ^F♭	E♯ vvF	^E♯ vF	^^E♯ **F**		
63 ♯ = 7	**D** ^3E♭♭	^D v3E♭	^^D vvE♭	^3D vE♭	v3D♯ E♭ ^3F♭♭	vvD♯ ^E♭ v3F♭	vD♯ ^^E♭ vvF♭	D♯ ^3E♭ vF♭	^D♯ v3E F♭	^^D♯ vvE ^F♭	^3D♯ vE ^^F♭	v3D𝗫 **E** ^3F♭	^E v3F	^^E vvF	^3E vF	v3E♯ **F**			
64 ♯ = 3	**D** vE♭3	^D E♭3	vD♯ ^E♭3	D♯ vE♭♭	^D♯ E♭♭ vF♭4	vD𝗫 ^E♭♭ F♭4	D𝗫 vE♭ ^F♭4	^D𝗫 E♭ vF♭3	vD♯3 ^E♭ F♭3	D♯3 vE ^F♭3	^D♯3 **E** vF♭♭	vD♯4 ^E F♭♭	D♯4 vE♯ ^F♭♭	^D♯4 E♯ vF♭	^E♯ F♭	vE𝗫 ^F♭	E𝗫 vF	^E𝗫 **F**	
65 ♯ = 6	**D** ^E♭♭	^D ^^E♭♭	^^D v3E♭	^3D vvE♭	vvD♯ vE♭ F♭♭	vD♯ E♭ ^F♭♭	D♯ ^E♭ ^^F♭♭	^D♯ ^^E♭ v3F♭	^^D♯ v3E vvF♭	^3D♯ vvE vF♭	vvD𝗫 vE F♭	vD𝗫 **E** ^F♭	D𝗫 ^E ^^F♭	^^E v3F	^3E vvF	vvE♯ vF	vE♯ **F**		
66 ♯ = 9	**D** v3E♭	^D vvE♭	^^D vE♭	^3D E♭ v3F♭	^4D ^E♭ vvF♭	v4D♯ ^^E♭ vF♭	v3D♯ ^3E♭ F♭	vvD♯ ^4E♭ ^F♭	vD♯ v4E ^^F♭	D♯ v3E ^3F♭	^D♯ vvE ^4F♭	^^D♯ vE v4F	^3D♯ **E** v3F	^E vvF	^^E vF	^3E **F**			
67 ♯ = 5	**D** vE♭♭	^D E♭♭	^^D ^E♭♭	vvD♯ ^^E♭♭ ^F♭3	vD♯ vvE♭ ^^F♭3	D♯ vE♭ vvF♭♭	^D♯ E♭ vF♭♭	^^D♯ ^E♭ F♭♭	vvD𝗫 ^^E♭ ^F♭♭	vD𝗫 vvE ^^F♭♭	D𝗫 vE vvF♭	^D𝗫 **E** vF♭	^^D𝗫 ^E F♭	vvD♯3 ^^E ^F♭	vD♯3 vvE♯ ^^F♭	vE♯ vvF	E♯ vF	^E♯ **F**	
68 ♯ = 8	**D** v4E♭	^D v3E♭	^^D vvE♭	^3D vE♭ ^3F♭♭	^4D E♭ v4F♭	v3D♯ ^E♭ v3F♭	vvD♯ ^^E♭ vvF♭	vD♯ ^3E♭ vF♭	D♯ v4E F♭	^D♯ v3E ^F♭	^^D♯ vvE ^^F♭	^3D♯ vE ^3F♭	^4D♯ **E** v4F	v3D𝗫 ^E v3F	^^E vvF	^3E vF	^4E **F**		
69 ♯ = 4	**D** ^E♭3	^D vvE♭♭	^^D vE♭♭	vD♯ E♭♭	D♯ ^E♭♭ vvF♭3	^D♯ vvE♭ vF♭3	^^D♯ vE♭ F♭3	vD𝗫 E♭ ^F♭3	D𝗫 ^E♭ vvF♭♭	^D𝗫 vvE vF♭♭	^^D𝗫 vE F♭♭	vD♯3 **E** ^F♭♭	D♯3 ^E vvF♭	^D♯3 ^^E vF♭	^^D♯3 vE♯ F♭	E♯ ^F♭	^E♯ vvF	^^E♯ vF	vE𝗫 **F**
70 ♯ = 7	**D** ^^E♭♭	^D ^3E♭♭	^^D v3E♭	^3D vvE♭ F♭♭	v3D♯ vE♭ ^F♭♭	vvD♯ E♭ ^^F♭♭	vD♯ ^E♭ ^3F♭♭	D♯ ^^E♭ v3F♭	^D♯ ^3E♭ vvF♭	^^D♯ v3E vF♭	^3D♯ vvE F♭	v3D𝗫 vE ^F♭	vvD𝗫 **E** ^^F♭	vD𝗫 ^E ^3F♭	D𝗫 ^^E v3F	^3E vvF	v3E♯ vF	vvE♯ **F**	
71 ♯ = 10	**D** v3E♭	^D vvE♭	^^D vE♭	^3D E♭ v3F♭	^4D ^E♭ vvF♭	^5D ^^E♭ vF♭	v4D♯ ^3E♭ F♭	v3D♯ ^4E♭ ^F♭	vvD♯ v5E ^^F♭	vD♯ v4E ^3F♭	D♯ v3E ^4F♭	^D♯ vvE v5F	^^D♯ vE v4F	^3D♯ **E** v3F	^E vvF	^^E vF	^3E **F**		
72 ♯ = 6	**D** E♭♭	^D ^E♭♭	^^D ^^E♭♭	^3D v3E♭	vvD♯ vvE♭ vvF♭♭	vD♯ vE♭ vF♭♭	D♯ E♭ F♭♭	^D♯ ^E♭ ^F♭♭	^^D♯ ^^E♭ ^^F♭♭	^3D♯ v3E v3F♭	vvD𝗫 vvE vvF♭	vD𝗫 vE vF♭	D𝗫 **E** F♭	^D𝗫 ^E ^F♭	^^D𝗫 ^^E ^^F♭	^3E v3F	vvE♯ vvF	vE♯ vF	E♯ **F**
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

Note names may be affected by the chord they are part of. In 24-edo, an ^C minor chord may well be written ^C ^E♭ ^G, not ^C vE ^G. This chart shows all the options for 24-edo in full:

Table 2.3 – Notation Guide for 24-edo Using Ups and Downs, In Full

24-edo	diatonic sharp-2	**D** vvD♯ E♭♭ vvE♭	^D vD♯ ^E♭♭ vE♭	^^D D♯ vvDˣ ^^E♭♭ E♭ vvE F♭♭ vvF♭	^D♯ vDˣ ^E♭ vE ^F♭♭ vF♭	^^D♯ **D**ˣ ^^E♭ **E** vvE♯ ^^F♭♭ F♭ vvF	^E vE♯ ^F♭ vF	^^E E♯ ^^F♭ **F**

Of all the diatonic edos, 42-edo has the sharpest fifth, and 47-edo has the flattest. 42-edo requires the use of trups and truds, and 47-edo requires triple sharps and flats. In certain keys, even more. For example, in 47-edo, a 4:5:6:7 chord three edosteps above D requires at least quintuple sharps or flats to spell correctly.

Extremely large edos require even more ups and downs. For example, in 1200bbb-edo, a sharp-100 edo, an up equals exactly one cent, and the notation is the same as simply writing the cents offset from 12-edo next to each note, with an up or down replacing the plus or minus sign.

The Tonal Plexus by H-Pi is a 205-edo keyboard, with the keys visually grouped into 41 blocks of five. 205-edo is a sharp-20 edo. To notate the Tonal Plexus, it would make sense to borrow lifts and drops (/ and \) from rank-2 pergen notation, and use an extended 41edo notation, such that /^{5}1 = ^1 = 1\41.

Section 3 – Chord Names and Scale Names

Chord names are based on jazz chord names (see "A Player's Guide to Chords & Harmony" by Jim Aiken), with ups and downs added in. Unlike JI chords, enharmonic substitutions are allowed. Where jazz has flat-9, classical has minor-9. Both ways of naming intervals are widely used, so both ways are presented here. A few special cases:

In perfect edos (7, 14, 21, 28 and 35), every interval is perfect. When naming chords, never use major, minor, dim, aug, sharp or flat. Substitute up for upmajor and upminor, and down for downmajor and downminor.

Supersharp edos: 13-edo and 18-edo use their second-best fifth, to convert them to superflat. 6-edo is notated as a subset of 12-edo, and 8-edo as a subset of 24-edo.

In superflat edos (9, 11, 13b, 16, 18b and 23), major is narrower than minor, and sharpening or augmenting lowers the pitch.

All lower degrees are assumed to be present: A 7th chord has a 3rd and a 5th, a 9th chord has both these plus a 7th, an 11th chord has all these plus a 9th, and a 13th chord has all these plus an 11th. This allows easy naming of a chord that uses the first 13 harmonics. A 13th chord without a 9th or 11th can optionally be named as an add-6 chord.

An up or down (or dup, etc.) in the chord name immediately after the root raises or lowers the 3rd, and also the 6th, 7th or 11th, if present. Thus "G down-nine" is the usual G9 chord with the 3rd and 7th lowered: Gv9 = G–vB–D–vF–A. Likewise, G~11 has root, ~3, P5, ~7, M9 and ~11.

The rationale for this rule is that a chord often has a note a perfect fourth or fifth above the 3rd. Furthermore, in many edos, the upfifth, downfifth, upfourth and downfourth will all be quite dissonant and rarely used in chords. Thus if the 3rd is upped or downed, the 6th or 7th likely would be too. If the 7th is, the 11th would be too. However, the 9th likely wouldn't, since that would create an upfifth or a downfifth with the 5th. Nor would the 13th, in order to make a good fifth with the 9th. Mnemonic: every other note of a stacked-3rds chord is affected: 6th – root – 3rd – 5th – 7th – 9th – 11th – 13th.

Alterations are always enclosed in parentheses. Thus a dom7 flat-5 chord is C7(♭5) not C7♭5. Additions never use parentheses. In the written name, commas are used as needed to separate added notes. Thus G–B–D–vF is written G,v7. **A comma that immediately follows the chord root must be spoken as "add".** Thus G,v7 is "G add down-seven". "Add" is otherwise spoken as needed, and never written. "Sus" is rarely spoken and never written.

An alteration such as "up-three" makes the 3rd either upmajor or upminor, depending on the context. For example, C7(^3) has an upmajor 3rd, but Cm7(^3) has an upminor 3rd, and C~7(^3) has an upmid 3rd. Without context, as with added notes, the usual assumptions are made: M2, M3, P4, P5, M6, m7, M9, P11, M13. Thus C,^7 has an upminor 7th.

In Cvm9(^7), the (^7) changes the vB♭ to an ^B♭. Thus the (^7) cancels all implied ups or downs before adding an up. To change the vB♭ to a plain B♭, avoid Cvm9(m7) or Cvm9(♭7), instead write Cm9(v3). (But see 11th chords below.)

Chord progressions use ups/downs notation to name the roots. Here's the first four chords of Paul Erlich's 22-edo composition "Tibia", which is written out in Section 4:

GvM7no5 = G downmajor seven, no five
^E♭v,9 = up E-flat down, add nine
C7(4) = C-seven four
Av,7 = A down add-seven

Relative notation applies ups and downs to the usual roman numerals. **Roman numerals must always be uppercase** because otherwise both VIIm and vIIm would be written as vii. The "Tibia" chords:

IvM7no5 = one downmajor seven, no five
^♭VIv,9 = upflat-six down, add nine
IV7(4) = four-seven sus-four
IIv,7 = two down add-seven

Conventionally, when the key is minor, sometimes the roman numerals default to a minor scale, and Am – C – G – Am might be written i – III – VII – i. This can create confusion if the song modulates from minor to the parallel major (or

vice versa), or if the tonic chord has a neutral 3rd. Thus Am – C – G – Am is better written as Im – ♭III – ♭VII – I.

To find a chord's name, determine its component intervals, then use the following tables. These tables aren't exhaustive, but they do provide enough examples to extrapolate from.

If ups and downs are removed from the name, the result should be the closest conventional 12-edo chord. Therefore, in sharp-3 edos, avoid intervals with dups and duds: upflat-five is preferred to dud-five.

Chord names are mostly independent of the edo: A–^C–E in most edos is A^m or A upminor. But in perfect edos, major, minor, aug and dim aren't used, and A–^C–E is A^ or A up. And in edos with a sharpness of 2, 4, or higher, mid, up-mid, etc. are used. In sharp-2 edos, A–^C–E is A~ or A mid. There are extra columns in the tables below that cover perfect edos and edos with large sharpness. If there is no entry in this column, use the 2nd and 3rd columns instead.

Table 3.1 – Various triads

Chord	Written name	Spoken name	In perfect edos	In certain other edos
C E G	C	C or C major	C or C perfect	
C ^E G	C^	C up or C upmajor	C up	
C vE G	Cv	C down or C downmajor	C down	In sharp-2 edos: C~ = C mid
C vvE G	Cvv	C dud or C dudmajor	C dud	In sharp-4 edos: C~ = C mid In sharp-5 or sharp-6 edos: C^~ = C upmid

This table shows how altering the 3rd or the 5th affects the name of the triad. The conventional abbreviations for aug and dim are "+" and "o". These cryptic symbols can be replaced with the far more obvious and intuitive "a" and "d".

Table 3.2 – Various triads with altered 3rd and/or 5th

	major C E G	**minor** C E♭ G	**sus4** C F G	**sus2** C D G	**augmented** C E G♯		**diminished** C E♭ G♭	
what's downed								
nothing	C	Cm	C4	C2	Ca	C+	Cd	C^{o}
3rd	Cv	Cvm	Cv4	Cv2	Cva	Cv+	Cvd	Cv^{o}
5th	C(v5)	Cm(v5)	C4(v5)	C2(v5)	Ca(v5)	C+(v5)	Cd(v5)	C^{o}(v5)
3rd, 5th	Cv(v5)	Cvm(v5)	Cv4(v5)	Cv2(v5)	Cva(v5)	Cv+(v5)	Cvd(v5)	Cv^{o}(v5)

Although based on jazz chord names, these chord names are meant for all genres, not just jazz. Thus Cd is a triad, not a tetrad. A diminished tetrad is a d7 chord.

In some edos, it's common to have chords containing dim 4ths, dim 3rds, aug 7ths, dub-dim 5ths, etc. Examples are given in each table of such extreme chords. A chord name can never begin with "sharp" or "flat", because then "C sharp two" could mean either C D♯ G or C♯ D♯ G♯. Thus aug/dim is better than sharp/flat. If you must use sharp or flat to start a chord name, use "sus" as well, even though the note may not actually be suspended.

Table 3.3 – Various unusual triads

Chord	Written name		Spoken name		In perfect edos
C F♭ G	C(♭4)	C(d4)	C sus-flat-four	C dim-four	C4
C E♯ G	C(♯3)	C(a3)	C sus-sharp-three	C aug-three	C
C E♭♭ G	C(♭♭3)	C(d3)	C sus-dub-flat-three	C dim-three	C
C D♯ G	C(♯2)	C(a2)	C sus-sharp-two	C aug-two	C2
C vE♭ F G	Cvm,4		C downminor add-four (6:7:8:9 in 31edo)		Cv,4 = C down add four

Chord	Written name	Spoken name	In perfect edos
C E♭ ^F G	Cm,^4	C minor add-up-four	C,^4 = C add up-four
C D E G	C,2	C add-two	
C G	C5	C five	
C E	Cno5	C no-five (never C3, too similar to JI chords C3o and C3u)	

Table 3.4 – Triads with altered fifths, showing both styles of chord names

Chord notes	Written name		Spoken name	
C E G♭	C(♭5)	C(d5)	C major flat-five	C dim-five
C E♭ G♭♭	Cm(♭♭5)	Cm(dd5)	C minor dub-flat-five	C minor dub-dim five
C E♭♭ G♭	Cd(♭♭3)	Cd(d3)	C dim dub-flat-three	C dim dim-three
C E♭♭ G♭♭	C(♭♭3,♭♭5)	C(d3,dd5)	C sus-dub-flat-three dub-flat-five	C dim-three dub-dim five
C E♭ G♯	Cm(♯5)	Cm(a5)	C minor sharp-five	C minor aug-five
C E♯ G♯	Ca(♯3)	Ca(a3)	C aug sharp-three	C aug aug-three
C E G♯♯	C(♯♯5)	C(aa5)	C major dub-sharp-five	C dub-aug-five

Seventh chords: Those in which the 7th is not a perfect 5th or a diminished 5th above the 3rd are generally named as "add-seven" chords.

Table 3.5 – Dom7 chords with altered 3rd and/or 7th

Chord notes	Written	Spoken
C E G B♭	C7	C seven
C vE G B♭	Cv,7	C down add-seven
C E G vB♭	C,v7	C add down-seven
C vE G vB♭	Cv7	C down-seven

In 19-edo, the 4:5:6:7 chord is C–E–G–B♭♭. This is C,d7= C add dim-seven, or C,♭♭7 = C add dub-flat-seven. dub-flat because sharp and flat are relative to M7, the default 7th in a scale. One might wonder, why M7 and not m7, since m7 is the default 7th in a chord? C–E–G–B♭♭ would then be C,♭7 = C add flat-seven. The problem is that in the key of D, C–E–G–B♭♭ would be ♭VII,♭7 = flat-seven add flat-seven. The root of the chord would be ♭VII = min 7th, but the 7th of the chord would be ♭7 = dim 7th, and "flat-seven" would have two different meanings, very confusing. So a dim 7th is written as d7 or ♭♭7, but a minor 7th is still 7, and a major 7th is still M7.

Allowing add-7 chords means that the wordy "minor-7 flat-5" and the confusing "half-dim" can be replaced with "dim add-7", written as Cd,7. Half-dim is especially confusing because any edo with an even sharpness will have half-diminished 5ths, 2nds, etc.

The next table shows how altering the 3rd, 5th and/or 7th affects the name of the chord. A comma must be spoken as "add", except for Cvd,d7.

Table 3.6 – Various seventh chords with altered 3rd, 5th and/or 7th (**bold** = the comma is spoken as "add")

what's downed	**maj7** C E G B	**dom7** C E G B♭	**min7** C E♭ G B♭	**dim-add-7** or **min7(♭5)** or **half-dim** C E♭ G♭ B♭			**dim7** C E♭ G♭ B♭♭	
nothing	CM7	C7	Cm7	**Cd,7**	Cm7(♭5)	C^{ø}	Cd7	C^{o}7
3rd	**Cv,M7**	**Cv,7**	**Cvm,7**	**Cvd,7**	**Cvm,7(♭5)**	C^{ø}(v3)	Cvd,d7	Cv^{o}d7
5th	CM7(v5)	C7(v5)	Cm7(v5)	Cd(v5)7	Cm7(v♭5)	C^{ø}(v5)	Cd7(v5)	C^{o}7(v5)
7th	**C,vM7**	**C,v7**	Cmv7	Cdv7	Cmv7(♭5)	C^{ø}(v7)	Cdvd7	C^{o}vd7
3rd, 5th	Cv(v5)M7	Cv(v5)7	Cvm(v5)7	Cvd(v5)7	**Cvm,7(v♭5)**	C^{ø}(v3v5)	Cvd(v5)d7	Cv^{o}(v5)d7
3rd, 7th	CvM7	Cv7	Cvm7	Cvdv7	Cvm7(♭5)	Cv^{ø}	Cvd7	Cv^{o}7
5th, 7th	C(v5)vM7	C(v5)v7	Cm(v5)v7	Cd(v5)v7	Cmv7(v♭5)	C^{ø}(v5v7)	Cd(v5)vd7	C^{o}(v5)vd7
3rd, 5th, 7th	CvM7(v5)	Cv7(v5)	Cvm7(v5)	Cvd(v5)v7	Cvm7(v♭5)	Cv^{ø}(v5)	Cvd7(v5)	Cv^{o}7(v5)

Table 3.7 – Various unusual seventh chords

Chord notes	Written name	Spoken name	In sharp-2 edos
C vE G vvB♭	Cv,vv7	C down dud seven	
C vE G ^B♭	Cv,^7	C down up-seven	C~7, C mid-seven
C E♯ G B♭	C7(♯3) or C7(a3)	C seven sharp-three or C seven aug-three	
C E G B♭♭	C,♭♭7 or C,d7	C add dub-flat-seven or C add dim-seven	
C E♭♭ G B♭♭	C(♭♭3)♭♭7 or C(d3)d7	C sus-dub-flat-three dub-flat-seven or C dim-three dim-seven	
C E G B♯	C,♯7 or C,a7	C add sharp-seven or C add aug-seven	
C E G C♭	C,♭8 or C,d8	C add flat-eight or C add dim-eight	

Sixth chords: if the 6th is not a perfect 4th or an augmented 4th above the 3rd, it's an "add-6" chord.

Table 3.8 – Various sixth and seventh chords with altered 3rd, 5th and/or 7th (**bold** = pronounce "," as add)

what's downed	**dom7sus4** C F G B♭		**aug7** C E G♯ B♭	**aug-maj7** C E G♯ B	**min-maj** C E♭ G B	**maj6** C E G A	**min6** C E♭ G A	**add-min6** C E G A♭
nothing	C7(4)	C4,7	Ca7	CaM7	CmM7	C6	Cm6	**C,m6**
3rd	C7(v4)	C(v4),7	**Cva,7**	Cva,M7	CvmM7	**Cv,6**	**Cvm,6**	**Cv,m6**
5th	C7(4v5)	C4(v5)7	Ca7(v5)	Ca(v5)M7	Cm(v5)M7	C6(v5)	Cm6(v5)	C(v5)m6
7th	Cv7(4)	C4v7	Cav7	Ca,vM7	CmvM7	**C,v6**	Cmv6	**C,vm6**
3rd, 5th	C7(v4v5)	C(v4v5)7	Cva(v5)7	Cva(v5)M7	Cvm(v5)M7	Cv(v5)6	Cvm(v5)6	Cv(v5)m6
3rd, 7th	Cv7(v4)	C(v4)v7	Cva7	Cva,vM7	CvmvM7	Cv6	Cvm6	Cv,vm6
5th, 7th	Cv7(4)v5	C4(v5)v7	Ca(v5)v7	Ca(v5)vM7	Cm(v5)vM7	C(v5)v6	Cm(v5)v6	C(v5)vm6
3rd, 5th, 7th	Cv7(v4v5)	C(v4v5)v7	Cva7(v5)	Cva(v5)vM7	Cvm(v5)vM7	Cv6(v5)	Cvm6(v5)	Cv(v5)vm6

Table 3.9 – Various unusual sixth chords

Chord notes	Written name	Spoken name	In sharp-2 edos
C ^E♭ G vA	C^m,v6	C upminor down-six	C~6, C mid-six
C E G A♭	C,♭6 or C,m6	C add flat-six or C add minor-six	
C E♭ G A♭	Cm,♭6 or Cm,m6	C minor flat-six or C minor minor-six	
C E♭ G A♯	Cm,♯6 or Cm,a6	C minor sharp-six or C minor aug-six	
C E♭♭ G A	C6(♭♭3) or C6(d3)	C six dub-flat three or C six dim-three	

Ninth chords: In **bolded** chords, the comma is spoken as "add". Double alterations need only a single pair of parentheses, e.g. C vE vG B D is named CM9(v3v5). Double additions need only a single comma, e.g. C E G vB vD is named C,vM7v9. But certain 6/9 chords require two commas. In these chords, marked with an asterisk *, only the first comma is spoken as "add".

Table 3.10 – Ninth chords

	add9	**maj9**	**dom9**	**min9**	**dom7♭9**	**maj6/9**	**min6/9**
what's downed	C E G D	C E G B D	C E G B♭ D	C E♭ G B♭ D	C E G B♭ D♭	C E G A D	C E♭ G A D
nothing	**C,9**	CM9	C9	Cm9	C7,♭9	C6,9	Cm6,9
3rd	**Cv,9**	CM9(v3)	C9(v3)	Cm9(v3)	**Cv,7♭9**	**Cv,6,9** *	**Cvm,6,9** *
5th	**C,9(v5)**	CM9(v5)	C9(v5)	Cm9(v5)	C7(v5)♭9	C6(v5)9	Cm6(v5)9
7th	——	CM9(v7)	C9(v7)	Cm9(v7)	**C,v7♭9**	**C,v6,9** *	Cm,v6,9
9th	**C,v9**	CM7,v9	C7,v9	Cm7,v9	C7,v♭9	C6,v9	Cm6,v9
3rd, 5th	Cv(v5)9	CM9(v3v5)	C9(v3v5)	Cm9(v3v5)	Cv(v5)7♭9	Cv(v5)6,9	Cvm(v5)6,9
3rd, 7th	——	CvM9	Cv9	Cvm9	Cv7,♭9	Cv6,9	Cvm6,9
3rd, 9th	Cv,v9	**Cv,M7v9** or CM7(v3)v9	**Cv,7v9** or C7(v3)v9	**Cvm,7v9** or Cm7(v3)v9	**Cv,7v♭9** or C7(v3)v♭9	**Cv,6v9** or C6(v3)v9	**Cvm,6v9** or Cm6(v3)v9
5th, 7th	——	CM9(v5v7)	C9(v5v7)	Cm9(v5v7)	C(v5)v7♭9	C(v5)v6,9	Cm(v5)v6,9
5th, 9th	C(v5)v9	CM7(v5)v9	C7(v5)v9	Cm7(v5)v9	C7(v5)v♭9	C6(v5)v9	Cm6(v5)v9
7th, 9th	——	**C,vM7v9**	**C,v7v9**	Cm,v7v9	C,v7v♭9	**C,v6v9**	Cm,v6v9
3rd, 5th, 7th	——	CvM9(v5)	Cv9(v5)	Cvm9(v5)	Cv7(v5)♭9	Cv6(v5)9	Cvm6(v5)9
3rd, 5th, 9th	Cv(v5)v9	Cv(v5)M7v9 or CM7(v3v5)v9	Cv(v5)7v9 or C7(v3v5)v9	Cvm(v5)7v9 or Cm7(v3v5)v9	Cv(v5)7v♭9 or C7(v3v5)♭9	Cv(v5)6v9 or C6(v3v5)v9	Cvm(v5)6v9 or Cm6(v3v5)v9
3rd, 7th, 9th	——	CvM7,v9	Cv7,v9	Cvm7,v9	Cv7,v♭9	Cv6,v9	Cvm6,v9
5th, 7th, 9th	——	C(v5)vM7v9	C(v5)v7v9	Cm(v5)v7v9	C(v5)v7v♭9	C(v5)v6v9	Cm(v5)v6v9
everything	——	CvM7(v5)v9	Cv7(v5)v9	Cvm7(v5)v9	Cv7(v5)v♭9	Cv6(v5)v9	Cvm6(v5)v9

Eleventh and Thirteenth chords: the 11th defaults to perfect. Upping the entire chord ups the 11th too. In a mid-eleven or mid-thirteen chord, the 11th is a mid 11th. In sharp-2 edos, the C E G ^B♭ D ^F example could be named C~11(^3), but C~11(M3) is better, because upmid is more simply described as major.

Table 3.11 – Eleventh chords

Chord notes	Written name	Spoken name	In perfect edos	In sharp-2 edos
C E G B♭ D F	C11	C eleven		
C G B♭ D F	C11no3 or C9(4)	C eleven no-three or C nine four		
C vE G B♭ D F	C11(v3)	C eleven down-three		
C ^E G ^B♭ D F	C^9,11	C up-nine eleven		
C ^E G ^B♭ D ^F	C^11	C up-eleven		
C E G B♭ D ^F	C9^11	C nine up-eleven		C9~11
C vE G ^B♭ D ^F	C^11(v3)	C up-eleven down-three		C~11
C E G ^B♭ D ^F	C^11(M3)	C up-eleven major-three		C~11(M3)
C ^E G ^B♭ D F♯	C^9♯11 or ^C9a11	C up-nine sharp-eleven or C up-nine aug-eleven	C^9,11	

C ^E G ^B D ^F	C^M11	C upmajor-eleven	C^11	
C ^E G ^B D ^F♯	C^M9^♯11	C upmajor-nine upsharp-eleven	C^11	

C ^E♭ G ^B♭ D ^F	C^m11	C upminor-eleven	C^11	C~11

C E♯ G B♯ D F♯♯ (Cy11 in 33-edo)	C9(♯3♯7)♯♯11 or C9(a3a7)aa11	C nine sharp-three sharp-seven dub-sharp-eleven or C nine aug-three aug-seven dub-aug-eleven	C11	
C E♭♭ G B♭♭ D F♭ (Cg11 in 33-edo)	C9(♭♭3♭♭7)♭11 or C9(d3d7)d11	C nine dub-flat three dub-flat seven flat-eleven or C nine dim-three dim-seven dim-eleven	C11	

Table 3.12 – Thirteenth chords

Chord notes	Written name	Spoken name	In perfect edos	In sharp-2 edos
C E G B♭ D F A	C13	C thirteen		
C E G B♭ D A	C9,13	C nine thirteen		
C ^E G ^B♭ D ^F A	C^13	C up-thirteen		
C ^E G ^B♭ D F A	C13(^3^7)	C thirteen up-three up-seven		C13(^3,~7)

C ^E♭ G ^B♭ D ^F A	C^m13	C upminor-thirteen	C^13	C~13
C vE G vB D vF A	CvM13	C downmajor-thirteen	Cv13	C~13(v11)
C vE G vB D vF♯ A	CvM13(v♯11)	C downmajor-13 downsharp-11	Cv13	
C vE G vB D F♯ A	CvM13(♯11)	C downmajor-13 sharp-11	Cv13	

Slash chords: Here's Bob Dylan's "Simple Twist of Fate" in standard 12-edo notation, using slash chords to indicate the descending bass line:

D	/	/	/	D/C♯	/	/	/
D/C	/	/	/	G/B	/	/	/
Gm/B♭	/	/	/	D	A4/C♯	G/B	/
D/A	/	A7	/	D	/	/	/

This song in 22-edo or 41-edo might use downmajor and upminor chords. Ups and downs are simply added in:

Dv	/	/	/	Dv/vC♯	/	/	/
Dv/C	/	/	/	Gv/vB	/	/	/
G^m/^B♭	/	/	/	Dv	A4/vC♯	Gv/vB	/
Dv/A	/	Av7	/	Dv	/	/	/

How to write slash chords in relative notation with roman numerals? Using figured bass, the 1st inversion of the I triad is I6. But this conflicts with the jazz chord names we've been using, in which I6 is a tetrad. What if we translate the bass notes into roman numerals the same way we translate chord roots? For example V7 over the 4th of the scale would be V7/IV. Unfortunately, this conflicts with secondary chord notation, in which V7/IV means I7 resolving to IV.

The solution is to devise a new notation, called relative slash notation. It names the bass note relative to the chord root, as if it were an ordinary chord component.

Iv	/	/	/	Iv/vM7	/	/	/
Iv/7	/	/	/	IVv/3	/	/	/
IV^m/3	/	/	/	Iv	V4/v3	IVv/3	/
Iv/5	/	Vv7	/	Iv	/	/	/

In order to be compatible with conventional notation, there is a rule that if the number after the slash is greater than 7, the slash means "add". Thus IV6/9 still means IV6add9.

Iv/7 is spoken as "one down slash seven" or "one down over seven". Note that V/3 would be "five over three", which sounds like the frequency ratio 5/3. To avoid confusion, for such chords one can instead say "five over the three".

The bass note's quality is inherited from the chord. IV^m has an upminor 3rd, thus IV^m/3 means IV^m/^m3. If the bass note isn't present in the chord, e.g. a 7th under a triad, the quality defaults to the usual M2 M3 P4 P5 M6 m7. Thus Iv/7 has a minor 7th bass and Iv/v7 has a downminor 7th bass. If the bass note's quality differs from that of the corresponding chord note, it's written out explicitly. Thus a vM7 chord over a minor 7th is IvM7/m7, not IvM7/7. An augmented chord over a perfect 5th is Iaug/P5, and a major chord over a diminished 5th is I/d5 or I/♭5.

This new notation is very logical. Everything up to and including the chord root is relative to the tonic, and everything after the root is relative to the root. "/3" always indicates a 1st inversion, "/5" a 2nd inversion, and "/6" or "/7" a 3rd inversion. Omitting the chord root allows discussions of the sound of inversions in the abstract. For example, v/3 vs. ^m/3 compares the 1st inversion of the downmajor triad to that of the upminor triad. And v7/7 vs. v/v7 compares the downminor 7th being present or absent in the upper voices.

Since this new slash notation is so powerful, one might even want to use it with absolute notation:

Dv	/	/	/	Dv/vM7	/	/	/
Dv/7	/	/	/	Gv/3	/	/	/
G^m/3	/	/	/	Dv	A4/v3	Gv/3	/
Dv/5	/	Av7	/	Dv	/	/	/

Scale Names

Heptatonic scales are named as if they were chords with seven notes. As usual, major is assumed, a C scale is a C major scale. While chords need a highly condensed name that can be fit several times into a measure of sheet music, scale names needn't be as concise. The ^, v and ~ symbols aren't used in scale names, instead the words are spelled out.

Analogous to downing a chord, downing a scale lowers the 3rd, 6th and 7th. Likewise, the aug scale is a major scale with the 3rd, 6th and 7th augmented. Thus the 2nd is treated as a 9th. Furthermore, as with chord names, individual notes can be added, altered or omitted.

Table 3.13 – Heptatonic Scales

Scale notes	Written/spoken name	In perfect edos	In sharp-2 edos
C D E F G A B C	C or C major	C or C perfect	
C D vE F G vA vB C	C down or C downmajor	C down	C mid
C D vE F G A B C	C down-3		C mid-3
C D vE F G A vB C	C down plain-6		C mid major-6
C D E♭ F G A♭ B♭ C	C minor	C	
C D vE♭ F G vA♭ vB♭ B C	C downminor add-7	C down add-7	
C D ^E♭ F G ^A♭ ^B♭ C	C upminor	C up	C mid
C D ^E♭ F G A♭ B♭ C	C minor up-3	C up-3	C minor mid-3
C D ^E♭ F G ^A♭ B♭ C	C upminor plain-7	C up plain-7	C mid minor-7 (or flat-7)
C D E♭ F G A B♭ C	C dorian	C	
C D ^E♭ F G ^A ^B♭ C	C updorian	C up	same, or C mid upmajor-6
C D ^E♭ F G A ^B♭ C	C updorian plain-6	C up plain-6	C mid major-6
C D E F G A B♭ C	C mixolydian	C	
C D ^E F G ^A ^B♭ C	C upmixolydian	C up	same, or C upmajor mid-7
C D E♯ F G A♯ B♯ C	C aug	C	
C D E♯ F G A♯ B C	C aug major-7	C	
C D E𝄪 F G A𝄪 B𝄪 C	C dub-aug	C	
C D E♯3 F G A♯3 B♯3 C	C trip-aug	C	
C D E♭♭ F G A♭♭ B♭♭ C	C dim	C	
C D E♭3 F G A♭3 B♭3 C	C dub-dim	C	
C D E♭♭ F G A B♭♭ C	C dorian dim-3 dim-7	C	

The five modes of the pentatonic scale can be named similarly to the seven heptatonic modes. Alternatively, they can be named as thirdless or fifthless modes. The two thirdless modes are named after the upper half of the scale. Perfect edos have no major or minor, and thus no modes, and pentatonic scales can be named by their omitted degrees.

Table 3.14 – Pentatonic Scales

Scale notes	Written/spoken name	In perfect edos	In sharp-2 edos
C D E G A C	C pentatonic or C major pentatonic	C no-4 no-7	
C D F G A	C mixolydian pentatonic or C major thirdless pentatonic	C no-3 no-7	
C D F G B♭ C	C dorian pentatonic or C minor thirdless pentatonic	C no-3 no-6	
C E♭ F G B♭ C	C minor pentatonic	C no-2 no-6	
C E♭ F A♭ B♭ C	C phrygian pentatonic or C fifthless pentatonic	C no-2 no-5	

C D vE G vA C	C down-pentatonic	C down no-4 no-7	same, or C mid-pentatonic
C D vE G A C	C pentatonic down-3	C no-4 no-7 down-3	same, or C pentatonic mid-3

C vE♭ F G vB♭ C	C downminor pentatonic	C down no-2 no-6	
C ^E♭ F G ^B♭ C	C upminor pentatonic	C up no-2 no-6	same, or possibly C mid-minor pentatonic?
C ^E♭ F G B♭ C	C minor pentatonic up-3	C no-2 no-6 up-3	C minor pentatonic mid-3

The harmonic pentatonic scale is the edo's nearest approximation of a y,z7,9 chord = 8:9:10:12:14:16, and the subharmonic pentatonic is the nearest approximation of a g,r6,11 chord = 12/(12:10:9:8:7:6). Thus 41-edo has C D vE G vB♭ C and C ^E♭ F G ^A C.

JI considerations (see appendix for a guide to the color notation)

If two edos map g1 = 81/80 and r1 = 64/63 the same, 7-limit chord names will be similar. Especially if the sharpness is the same. For example, 15-edo and 22-edo both map g1 to 1 edostep and r1 to zero edosteps. 19-edo and 26-edo both map g1 to 0 and r1 to 1. The next table groups edos with the same mappings into the same column:

Table 3.15 – Examples of various 7-limit JI chords in various edos

JI chord	12-edo (sharp-1)	15 & 22-edo (sharp-3)	19 & 26-edo (sharp-1)	21 & 28-edo (sharp-0)	24, 31 & 38 (sharp-2)	34, 41 & 48 (sharp-4) 46 & 53 (sharp-5)	72-edo (sharp-6)
Cy6	CM6	CvM6	CM6	C^6	CM6	CvM6	CvM6
Cy7	CM7	CvM7	CM7	C^7	CM7	CvM7	CvM7
Cg7	Cm7	C^m7	Cm7	Cv7	Cm7	C^m7	C^m7
Cz7	Cm7	Cm7	C(d3)d7	Cv7	vCm7	Cvm7	Cvvm7
Cr6	C6	C6	C(♯3)♯6	C^6	C^6	C^6	C^^6
Cy,z7,9	C9	C9(v3)	C9(d7)	C^,v7,^9	C9(v7)	Cv9	C9(v3,vv7)
Cg,r6,11	Cm6,11	Cm6,11(^3)	Cm,♯6,11	Cv,^6,11	Cm,^6,11	C^m6,11	Cvm,^^6,11
Cz,y6	Cm6	Cm,^6	C6(d3)	Cv,^6	Cm6(v3)	Cv6	Cvvm,v6
Cr,g7,9	C9	C9(^7)	C9(♯3)	C^,v7,^9	C9(^3)	C^9	C9(^^3,^7)
Cg7(zg5)	Cm7(♭5)	C^m7(^♭5)	Cm7(♭♭5)	Cv7(vv5)	Cm7(v♭5)	C^m7(♭5)	C^m7(v♭5)
Cz7(zg5)	Cm7(♭5)	Cm7(^♭5)	C(d3,dd5,d7)	Cv7(vv5)	Cvm7(v♭5)	Cvm7(♭5)	Cvvm7(v♭5)

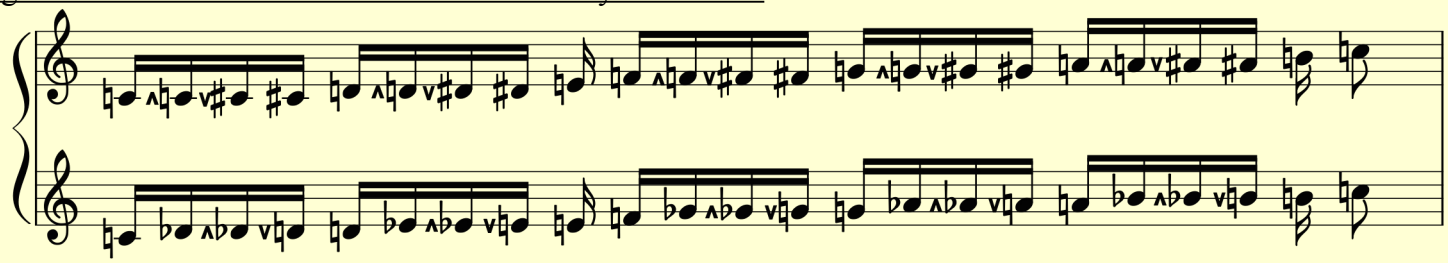
Translating a JI chord to an edo can be tricky. In 21-edo and 28-edo, 9/4 is best approximated by an upmajor 9th. Approximating each note individually, Cy,z7,9 = 4:5:6:7:9 becomes C^,v7,^9. But this chord would sound better with a major 9th, avoiding a dissonant upfifth between the 5th and the 9th. However, no5 chords are better with an upmajor 9th, e.g. Cy,9no5.

In 21-edo and 28-edo, 3/2 plus 3/2 does not equal 9/4. Inequalities like this are inevitable. Every edo approximates a JI rung with a certain cents discrepancy. As JI intervals become more remote, the discrepancies accumulate, until they eventually total more than half an edostep. At this point, the best approximation of a JI interval is not equal to the sum of the best approximations of each component rung.

Section 4 – Staff Notation and Key Signatures

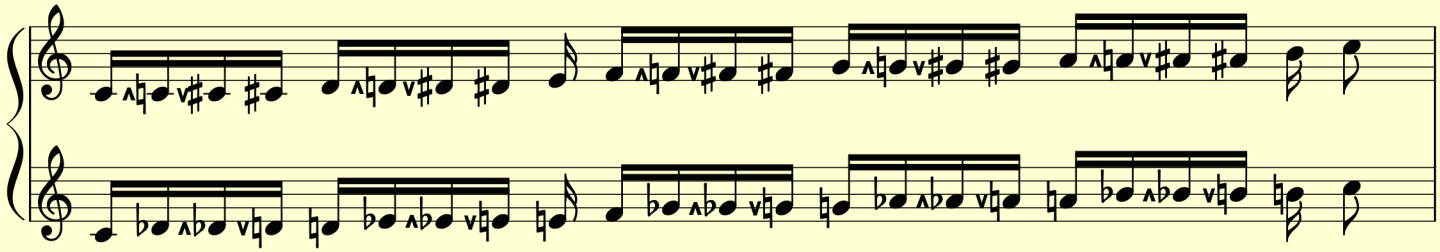
In staff notation, ups and downs, like sharps and flats, affect all successive notes in the same octave in the same measure. One approach is to always include both the up/down and the sharp/flat/natural for every note:

Figure 4.1 – 22-edo staff notation with mandatory accidentals



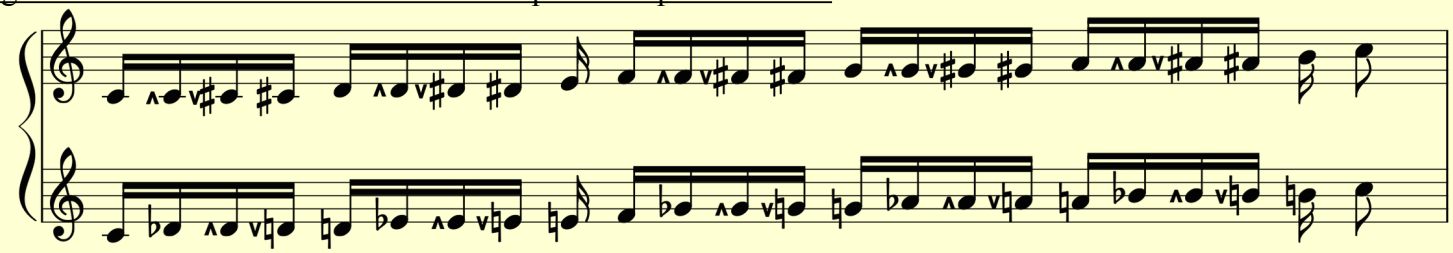
This approach is especially appropriate for atonal or highly chromatic music. A key signature isn't needed and is often omitted. For more tonal music, clutter can be reduced by using a key signature, and omitting accidentals implied by it.

Figure 4.2 – 22-edo staff notation with minimal accidentals (assumes a C major key signature)



Clutter can be reduced even more by using ups and downs independently of sharps and flats:

Figure 4.3 – 22-edo staff notation with independent ups and downs



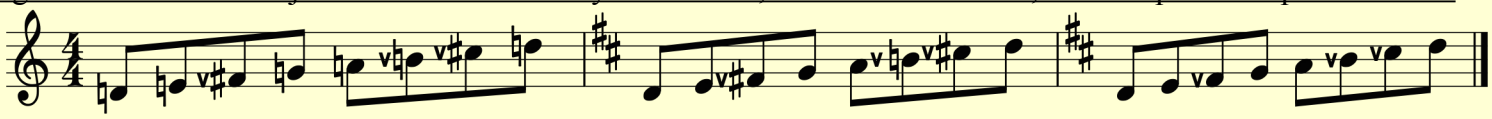
If an up or a down appears without a sharp or a flat, it does not cancel any implied sharp or flat. In the example above, the 3rd note in the lower staff is ^D♭. An implied sharp or flat must be explicitly cancelled with a natural sign, as with the 4th note in the lower staff.

However, a sharp, flat or natural without an up or a down does cancel any implied up or down. In fact, this is the only way to cancel an up or a down. Thus the 4th note in the upper staff is C♯. (It would be possible to have an additional accidental, a "plain sign", analogous to the natural sign, that cancels ups and downs without affecting sharps and flats.)

Trills can always be written as a 2nd, e.g. C–D♭ or ^C– ^D♭ or vC♯–vD or C♯–D.

One is free to use any of these methods, depending on the music. Here's a D downmajor scale using all three methods:

Figure 4.4 – D downmajor scale with mandatory accidentals, minimal accidentals, and independent ups and downs



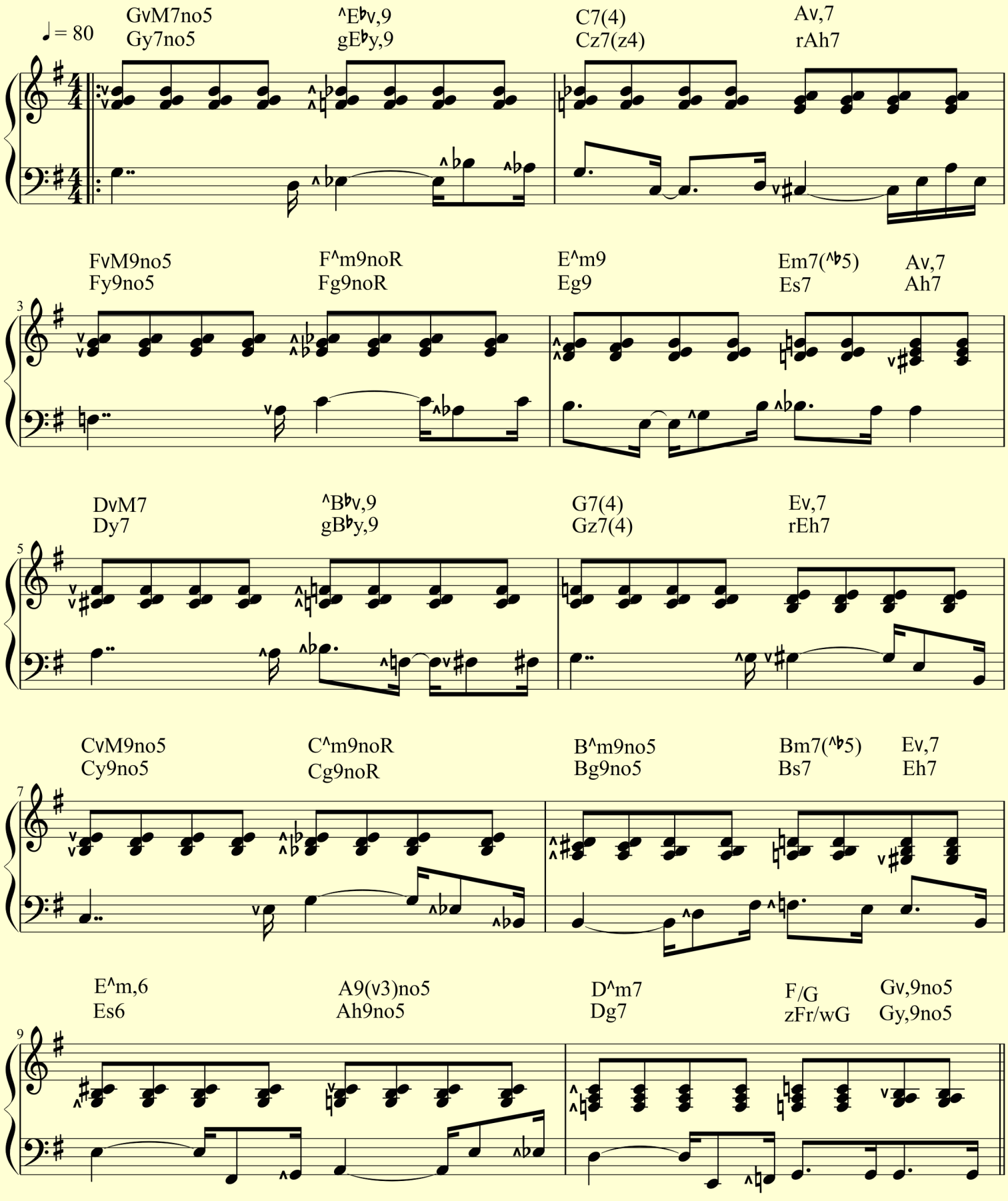
The mandatory accidental approach is clearly overkill in this example. It's better for more atonal pieces.

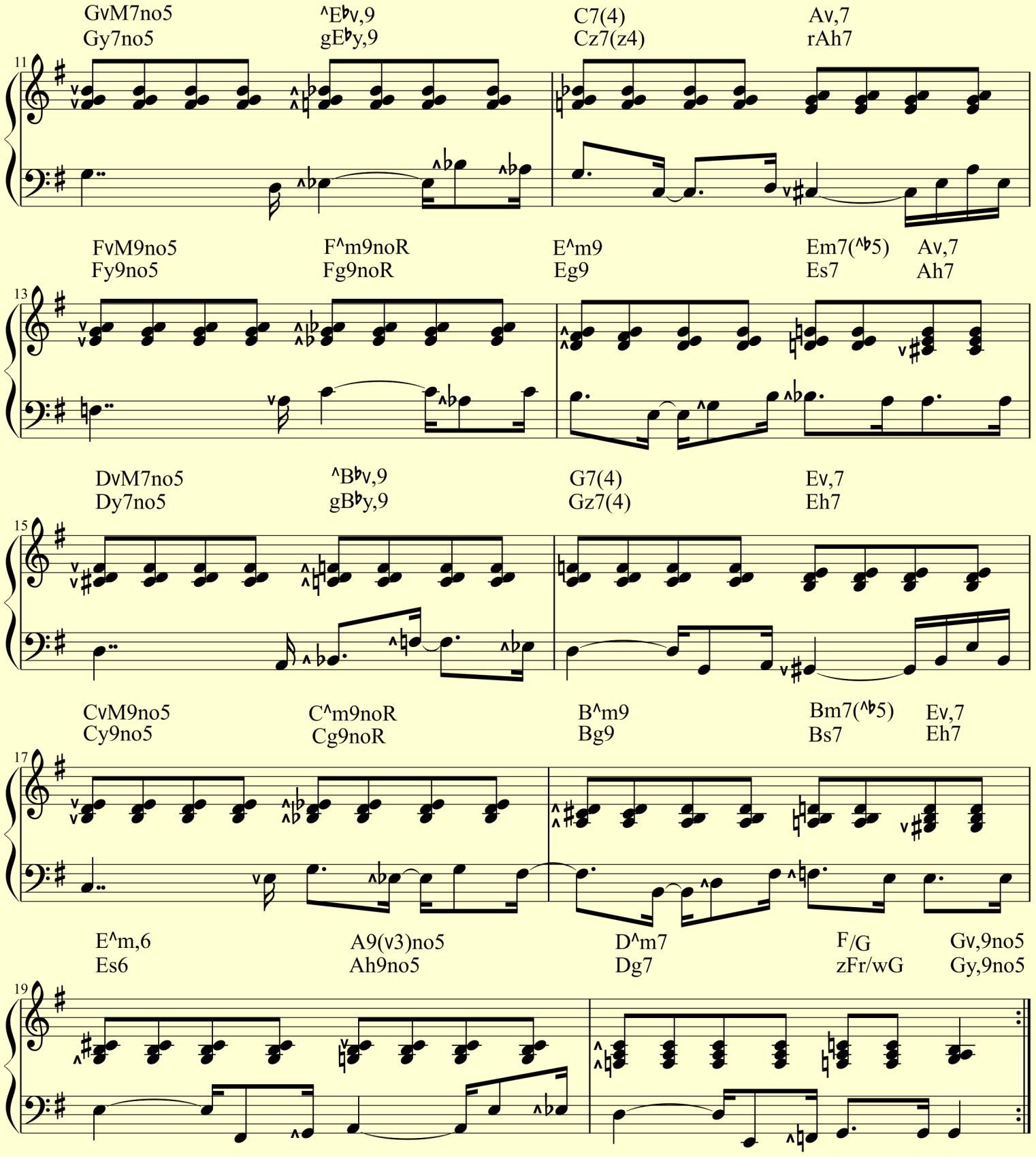
Paul Erlich's composition "Tibia" for 22-edo piano (www.TallKite.com/words/Tibia.mp3) is very chromatic, but also very tonal. The chords are written out in ups and downs notation and also in color notation (see the appendix). Independent ups and downs are used. Measure 9 has an example of a down not canceling a sharp.

Figure 4.5 – "Tibia" in G with independent ups and downs

Tibia in G in 22edo

Paul Erlich





Key Signatures: In any edo, any of the keys/frets can be the tonic of either a major scale or a minor scale. Each of the black keys has at least two names (e.g. in 12-edo, C♯ and D♭), and there are many possible keys, each with its own key signature. There follows a guide to edos 19, 22, 17, 24, 26, 27 and 31, as well as the perfect, pentatonic and superflat edos.

19-edo: There are 31 possible keys. However, if avoiding tonics that have double sharps and double flats, most keys have only one name. The only exceptions are the E♯/F♭ key and the B♯/C♭ key. For these two, the flat names are preferred for major keys, and the sharp ones for minor, in order to minimize double sharps and double flats in the key signature. E♯ and B♯ major, and F♭ and C♭ minor, are alternate keys that would only be used in special circumstances, for example if modulating from A♯ major to E♯ major, or from F♭ major to F♭ minor.

Table 4.1 – Preferred tonic names for 19-edo

major	C	C♯	D♭	D	D♯	E♭	E	F♭	F	F♯	G♭	G	G♯	A♭	A	A♯	B♭	B	C♭
minor	"	"	"	"	"	"	"	E♯	"	"	"	"	"	"	"	"	"	"	"	B♯

Table 4.2 – The 19-edo key signatures, in chain-of-fifths order

key signature	major key	major scale	minor key	minor scale
(𝄫 𝄫 𝄫 𝄫 ♭ ♭ ♭)			(F♭ minor)	(F♭ G♭ A𝄫 B𝄫 C♭ D𝄫 E𝄫 F♭)
(𝄫 𝄫 𝄫 ♭ ♭ ♭ ♭)			(C♭ minor)	(C♭ D♭ E𝄫 F♭ G♭ A𝄫 B𝄫 C♭)
𝄫 𝄫 ♭ ♭ ♭ ♭ ♭			G♭ minor	G♭ A♭ B𝄫 C♭ D♭ E𝄫 F♭ G♭
𝄫 ♭ ♭ ♭ ♭ ♭ ♭	F♭ major	F♭ G♭ A♭ B𝄫 C♭ D♭ E♭ F♭	D♭ minor	D♭ E♭ F♭ G♭ A♭ B𝄫 C♭ D♭
♭ ♭ ♭ ♭ ♭ ♭ ♭	C♭ major	C♭ D♭ E♭ F♭ G♭ A♭ B♭ C♭	A♭ minor	A♭ B♭ C♭ D♭ E♭ F♭ G♭ A♭
♭ ♭ ♭ ♭ ♭ ♭	G♭ major	G♭ A♭ B♭ C♭ D♭ E♭ F G♭	E♭ minor	E♭ F G♭ A♭ B♭ C♭ D♭ E♭
♭ ♭ ♭ ♭ ♭	D♭ major	D♭ E♭ F G♭ A♭ B♭ C D♭	B♭ minor	B♭ C D♭ E♭ F G♭ A♭ B♭
♭ ♭ ♭ ♭	A♭ major	A♭ B♭ C D♭ E♭ F G A♭	F minor	F G A♭ B♭ C D♭ E♭ F
♭ ♭ ♭	E♭ major	E♭ F G A♭ B♭ C D E♭	C minor	C D E♭ F G A♭ B♭ C
♭ ♭	B♭ major	B♭ C D E♭ F G A B♭	G minor	G A B♭ C D E♭ F G
♭	F major	F G A B♭ C D E F	D minor	D E F G A B♭ C D
no sharps or flats	C major	C D E F G A B C	A minor	A B C D E F G A
♯	G major	G A B C D E F♯ G	E minor	E F♯ G A B C D E
♯ ♯	D major	D E F♯ G A B C♯ D	B minor	B C♯ D E F♯ G A B
♯ ♯ ♯	A major	A B C♯ D E F♯ G♯ A	F♯ minor	F♯ G♯ A B C♯ D E F♯
♯ ♯ ♯ ♯	E major	E F♯ G♯ A B C♯ D♯ E	C♯ minor	C♯ D♯ E F♯ G♯ A B C♯
♯ ♯ ♯ ♯ ♯	B major	B C♯ D♯ E F♯ G♯ A♯ B	G♯ minor	G♯ A♯ B C♯ D♯ E♯ F G♯
♯ ♯ ♯ ♯ ♯ ♯	F♯ major	F♯ G♯ A♯ B C♯ D♯ E♯ F♯	D♯ minor	D♯ E♯ F♯ G♯ A♯ B C♯ D♯
♯ ♯ ♯ ♯ ♯ ♯ ♯	C♯ major	C♯ D♯ E♯ F♯ G♯ A♯ B♯ C♯	A♯ minor	A♯ B♯ C♯ D♯ E♯ F♯ G♯ A♯
X ♯ ♯ ♯ ♯ ♯ ♯	G♯ major	G♯ A♯ B♯ C♯ D♯ E♯ F𝄪 G♯	E♯ minor	E♯ F𝄪 G♯ A♯ B♯ C♯ D♯ E♯
X X ♯ ♯ ♯ ♯ ♯	D♯ major	D♯ E♯ F𝄪 G♯ A♯ B♯ C𝄪 D♯	B♯ minor	B♯ C𝄪 D♯ E♯ F𝄪 G♯ A♯ B♯
X X X ♯ ♯ ♯ ♯	A♯ major	A♯ B♯ C𝄪 D♯ E♯ F𝄪 G𝄪 A♯		
(X X X X ♯ ♯ ♯)	(E♯ major)	(E♯ F𝄪 G𝄪 A♯ B♯ C𝄪 D𝄪 E♯)		
(X X X X X ♯ ♯)	(B♯ major)	(B♯ C𝄪 D𝄪 E♯ F𝄪 G𝄪 A♯ B♯)		

The next table has the same information, with the keys sorted in melodic order. Each major key is on the same row as its relative minor (e.g. C major and A minor). Relative majors and minors have the same key signature, with one exception. A♯ major and G♭ minor have different key signatures, to avoid a tonic with a double sharp (F𝄪 minor) or a double flat (B♭♭ major).

Table 4.3 – The 19-edo key signatures, in melodic order

major key	major scale	key signature	minor key	minor scale
C major	C D E F G A B C	no sharps or flats	A minor	A B C D E F G A
C♯ major	C♯ D♯ E♯ F♯ G♯ A♯ B♯ C♯	♯ ♯ ♯ ♯ ♯ ♯ ♯	A♯ minor	A♯ B♯ C♯ D♯ E♯ F♯ G♯ A♯
D♭ major	D♭ E♭ F G♭ A♭ B♭ C D♭	♭ ♭ ♭ ♭ ♭	B♭ minor	B♭ C D♭ E♭ F G♭ A♭ B♭
D major	D E F♯ G A B C♯ D	♯ ♯	B minor	B C♯ D E F♯ G A B
D♯ major	D♯ E♯ F𝄪 G♯ A♯ B♯ C𝄪 D♯	𝄪 𝄪 ♯ ♯ ♯ ♯ ♯	B♯ minor (C♭ minor)	B♯ C𝄪 D♯ E♯ F𝄪 G♯ A♯ B♯ (C♭ D♭ E♭♭ F♭ G♭ A♭♭ B♭♭ C♭)
E♭ major	E♭ F G A♭ B♭ C D E♭	♭ ♭ ♭	C minor	C D E♭ F G A♭ B♭ C
E major	E F♯ G♯ A B C♯ D♯ E	♯ ♯ ♯ ♯	C♯ minor	C♯ D♯ E F♯ G♯ A B C♯
(E♯ major) F♭ major	(E♯ F𝄪 G𝄪 A♯ B♯ C𝄪 D𝄪 E♯) F♭ G♭ A♭ B♭♭ C♭ D♭ E♭ F♭	(𝄪 𝄪 𝄪 𝄪 ♯ ♯ ♯) ♭♭ ♭ ♭ ♭ ♭ ♭ ♭	D♭ minor	D♭ E♭ F♭ G♭ A♭ B♭♭ C♭ D♭
F major	F G A B♭ C D E F	♭	D minor	D E F G A B♭ C D
F♯ major	F♯ G♯ A♯ B C♯ D♯ E♯ F♯	♯ ♯ ♯ ♯ ♯ ♯	D♯ minor	D♯ E♯ F♯ G♯ A♯ B C♯ D♯
G♭ major	G♭ A♭ B♭ C♭ D♭ E♭ F G♭	♭ ♭ ♭ ♭ ♭ ♭	E♭ minor	E♭ F G♭ A♭ B♭ C♭ D♭ E♭
G major	G A B C D E F♯ G	♯	E minor	E F♯ G A B C D E
G♯ major	G♯ A♯ B♯ C♯ D♯ E♯ F𝄪 G♯	𝄪 ♯ ♯ ♯ ♯ ♯ ♯	E♯ minor (F♭ minor)	E♯ F𝄪 G♯ A♯ B♯ C♯ D♯ E♯ (F♭ G♭ A♭♭ B♭♭ C♭ D♭♭ E♭♭ F♭)
A♭ major	A♭ B♭ C D♭ E♭ F G A♭	♭ ♭ ♭ ♭	F minor	F G A♭ B♭ C D♭ E♭ F
A major	A B C♯ D E F♯ G♯ A	♯ ♯ ♯	F♯ minor	F♯ G♯ A B C♯ D E F♯
A♯ major	A♯ B♯ C𝄪 D♯ E♯ F𝄪 G𝄪 A♯	𝄪 𝄪 ♯ ♯ ♯ ♯ ♯	G♭ minor	G♭ A♭ B♭♭ C♭ D♭ E♭♭ F♭ G♭
B♭ major	B♭ C D E♭ F G A B♭	♭ ♭	G minor	G A B♭ C D E♭ F G
B major	B C♯ D♯ E F♯ G♯ A♯ B	♯ ♯ ♯ ♯ ♯	G♯ minor	G♯ A♯ B C♯ D♯ E♯ F G♯
(B♯ major) C♭ major	(B♯ C𝄪 D𝄪 E♯ F𝄪 G𝄪 A♯ B♯) C♭ D♭ E♭ F♭ G♭ A♭ B♭ C♭	(𝄪 𝄪 𝄪 𝄪 𝄪 ♯ ♯) ♭ ♭ ♭ ♭ ♭ ♭ ♭	A♭ minor	A♭ B♭ C♭ D♭ E♭ F♭ G♭ A♭

19-edo requires triple sharps and triple flats at times. For example, in E♭, the 7/5 is B trip-flat, written B♭3. If the music is atonal, the triple flat can be avoided by simply writing A. But if the music has recognizable chords, perhaps Iy,z7 – zIIIg,r6 – IVy,z7, triple accidentals in certain keys are unavoidable. (See the appendix for a guide to the color notation.)

Using edo chord names, this would be I(d7) – ♭♭IIIm(A6) – IV(d7). In E♭, it would be E♭(d7) – G♭♭m(A6) – A♭(d7), and the G♭♭ chord's notes would be G♭♭ – B♭3 – D♭♭ – E♭. Writing A would be misleading, because the note is clearly a minor 3rd from the root. The general rule is: ensure that the relative intervals within a chord are correct, even if it creates extreme accidentals. Writing the key as D𝄪 instead of E♭ won't help, because the Iy,z7 chord becomes D𝄪 – F♯3 – A𝄪 – C♯, and we've traded a triple flat for a triple sharp.

22-edo: If the key/fret is a natural note, the choice of key signature is easy. But many of the black keys have three or four names. For example, the key midway between G and A is either ^A♭, vG♯, F𝄪 or B♭♭. How to choose a name?

One approach is to not allow ups and downs in the tonic or in the key signature. The key names are not in order, so that B♯ is a higher key than C or D♭. Some tonics will have double sharps or flats. If there are two possible names, for example B♯ and E♭♭, choose the one that minimizes double-sharps and double-flats in both the key signature and the tonic. This results in slightly different choices for major vs. minor:

Table 4.4 – Preferred tonic names and key signatures for 22-edo, using double sharps and double flats

major	C	D♭	E♭♭	C♯	D	E♭	F♭	D♯	E	F	G♭	E♯	F♯	G	A♭	B♭♭	G♯	A	B♭	C♭	A♯	B
minor	"	"	B♯	"	"	"	"	"	"	"	"	"	"	"	"	F𝄪	"	"	"	"	"	"

Table 4.5 – 22-edo key signatures using double sharps and double flats, in chain-of-fifths order

key signature	major key	major scale	minor key	minor scale
𝄫 𝄫 𝄫 𝄫 ♭ ♭ ♭			F♭ minor	F♭ G♭ A♭♭ B♭♭ C♭ D♭♭ E♭♭ F♭
𝄫 𝄫 𝄫 ♭ ♭ ♭ ♭	E♭♭ major	E♭♭ F♭ G♭ A♭♭ B♭♭ C♭ D♭ E♭♭	C♭ minor	C♭ D♭ E♭♭ F♭ G♭ A♭♭ B♭♭ C♭
𝄫 𝄫 ♭ ♭ ♭ ♭ ♭	B♭♭ major	B♭♭ C♭ D♭ E♭♭ F♭ G♭ A♭ B♭♭	G♭ minor	G♭ A♭ B♭♭ C♭ D♭ E♭♭ F♭ G♭
𝄫 ♭ ♭ ♭ ♭ ♭ ♭	F♭ major	F♭ G♭ A♭ B♭♭ C♭ D♭ E♭ F♭	D♭ minor	D♭ E♭ F♭ G♭ A♭ B♭♭ C♭ D♭
♭ ♭ ♭ ♭ ♭ ♭ ♭	C♭ major	C♭ D♭ E♭ F♭ G♭ A♭ B♭ C♭	A♭ minor	A♭ B♭ C♭ D♭ E♭ F♭ G♭ A♭
♭ ♭ ♭ ♭ ♭ ♭	G♭ major	G♭ A♭ B♭ C♭ D♭ E♭ F G♭	E♭ minor	E♭ F G♭ A♭ B♭ C♭ D♭ E♭
♭ ♭ ♭ ♭ ♭	D♭ major	D♭ E♭ F G♭ A♭ B♭ C D♭	B♭ minor	B♭ C D♭ E♭ F G♭ A♭ B♭
♭ ♭ ♭ ♭	A♭ major	A♭ B♭ C D♭ E♭ F G A♭	F minor	F G A♭ B♭ C D♭ E♭ F
♭ ♭ ♭	E♭ major	E♭ F G A♭ B♭ C D E♭	C minor	C D E♭ F G A♭ B♭ C
♭ ♭	B♭ major	B♭ C D E♭ F G A B♭	G minor	G A B♭ C D E♭ F G
♭	F major	F G A B♭ C D E F	D minor	D E F G A B♭ C D
no sharps or flats	C major	C D E F G A B C	A minor	A B C D E F G A
♯	G major	G A B C D E F♯ G	E minor	E F♯ G A B C D E
♯ ♯	D major	D E F♯ G A B C♯ D	B minor	B C♯ D E F♯ G A B
♯ ♯ ♯	A major	A B C♯ D E F♯ G♯ A	F♯ minor	F♯ G♯ A B C♯ D E F♯
♯ ♯ ♯ ♯	E major	E F♯ G♯ A B C♯ D♯ E	C♯ minor	C♯ D♯ E F♯ G♯ A B C♯
♯ ♯ ♯ ♯ ♯	B major	B C♯ D♯ E F♯ G♯ A♯ B	G♯ minor	G♯ A♯ B C♯ D♯ E♯ F G♯
♯ ♯ ♯ ♯ ♯ ♯	F♯ major	F♯ G♯ A♯ B C♯ D♯ E♯ F♯	D♯ minor	D♯ E♯ F♯ G♯ A♯ B C♯ D♯
♯ ♯ ♯ ♯ ♯ ♯ ♯	C♯ major	C♯ D♯ E♯ F♯ G♯ A♯ B♯ C♯	A♯ minor	A♯ B♯ C♯ D♯ E♯ F♯ G♯ A♯
X ♯ ♯ ♯ ♯ ♯ ♯	G♯ major	G♯ A♯ B♯ C♯ D♯ E♯ F𝄪 G♯	E♯ minor	E♯ F𝄪 G♯ A♯ B♯ C♯ D♯ E♯
X X ♯ ♯ ♯ ♯ ♯	D♯ major	D♯ E♯ F𝄪 G♯ A♯ B♯ C𝄪 D♯	B♯ minor	B♯ C𝄪 D♯ E♯ F𝄪 G♯ A♯ B♯
X X X ♯ ♯ ♯ ♯	A♯ major	A♯ B♯ C𝄪 D♯ E♯ F𝄪 G𝄪 A♯	F𝄪 minor	F𝄪 G𝄪 A♯ B♯ C𝄪 D♯ E♯ F𝄪
X X X X ♯ ♯ ♯	E♯ major	E♯ F𝄪 G𝄪 A♯ B♯ C𝄪 D𝄪 E♯		

Table 4.6 – 22-edo key signatures using double sharps and double flats, in melodic order

major key	major scale	key signature	minor key	minor scale
C major	C D E F G A B C	no sharps or flats	A minor	A B C D E F G A
D♭ major	D♭ E♭ F G♭ A♭ B♭ C D♭	♭ ♭ ♭ ♭ ♭	B♭ minor	B♭ C D♭ E♭ F G♭ A♭ B♭
E♭♭ major	E♭♭ F♭ G♭ A♭♭ B♭♭ C♭ D♭ E♭♭	𝄫 𝄫 𝄫 ♭ ♭ ♭ ♭	C♭ minor	C♭ D♭ E♭♭ F♭ G♭ A♭♭ B♭♭ C♭
C♯ major	C♯ D♯ E♯ F𝄪 G♯ A♯ B♯ C♯	♯ ♯ ♯ ♯ ♯ ♯ ♯	A♯ minor	A♯ B♯ C♯ D♯ E♯ F𝄪 G♯ A♯
D major	D E F♯ G A B C♯ D	♯ ♯	B minor	B C♯ D E F♯ G A B
E♭ major	E♭ F G A♭ B♭ C D E♭	♭ ♭ ♭	C minor	C D E♭ F G A♭ B♭ C
F♭ major	F♭ G♭ A♭ B♭♭ C D♭ E♭ F♭	𝄫 ♭ ♭ ♭ ♭ ♭ ♭	D♭ minor	D♭ E♭ F♭ G♭ A♭ B♭♭ C D♭
D♯ major	D♯ E♯ F𝄪 G♯ A♯ B♯ C𝄪 D♯	X X ♯ ♯ ♯ ♯ ♯	B♯ minor	D♯ E♯ F𝄪 G♯ A♯ B♯ C𝄪 D♯
E major	E F♯ G♯ A B C♯ D♯ E	♯ ♯ ♯ ♯	C♯ minor	C♯ D♯ E F♯ G♯ A B C♯
F major	F G A B♭ C D E F	♭	D minor	D E F G A B♭ C D
G♭ major	G♭ A♭ B♭ C♭ D♭ E♭ F G♭	♭ ♭ ♭ ♭ ♭ ♭	E♭ minor	E♭ F G♭ A♭ B♭ C♭ D♭ E♭
E♯ major	E♯ F𝄪 G𝄪 A♯ B♯ C𝄪 D𝄪 E♯	X X X X ♯ ♯ ♯ 𝄫 𝄫 𝄫 𝄫 ♭ ♭ ♭	F♭ minor	F♭ G♭ A♭♭ B♭♭ C♭ D♭♭ E♭♭ F♭
F♯ major	F♯ G♯ A♯ B C♯ D♯ E♯ F♯	♯ ♯ ♯ ♯ ♯ ♯	D♯ minor	D♯ E♯ F♯ G♯ A♯ B C♯ D♯
G major	G A B C D E F♯ G	♯	E minor	E F♯ G A B C D E
A♭ major	A♭ B♭ C D♭ E♭ F G A♭	♭ ♭ ♭ ♭	F minor	F G A♭ B♭ C D♭ E♭ F
B♭♭ major	B♭♭ C♭ D♭ E♭♭ F♭ G♭ A♭ B♭♭	𝄫 𝄫 ♭ ♭ ♭ ♭ ♭	G♭ minor	G♭ A♭ B♭♭ C♭ D♭ E♭♭ F♭ G♭
G♯ major	G♯ A♯ B♯ C♯ D♯ E♯ F𝄪 G♯	X ♯ ♯ ♯ ♯ ♯ ♯	E♯ minor	E♯ F𝄪 G♯ A♯ B♯ C♯ D♯ E♯
A major	A B C♯ D E F♯ G♯ A	♯ ♯ ♯	F♯ minor	F♯ G A B C♯ D E F♯
B♭ major	B♭ C D E♭ F G A B♭	♭ ♭	G minor	G A B♭ C D E♭ F G
C♭ major	C♭ D♭ E♭ F♭ G♭ A♭ B♭ C♭	♭ ♭ ♭ ♭ ♭ ♭ ♭	A♭ minor	A♭ B♭ C♭ D♭ E♭ F♭ G♭ A♭
A♯ major	A♯ B♯ C𝄪 D♯ E♯ F𝄪 G𝄪 A♯	X X X ♯ ♯ ♯ ♯	F𝄪 minor	F𝄪 G𝄪 A♯ B♯ C𝄪 D♯ E♯ F𝄪
B major	B C♯ D♯ E F♯ G♯ A♯ B	♯ ♯ ♯ ♯ ♯	G♯ minor	G♯ A♯ B C♯ D♯ E F♯ G♯

A disadvantage of these key names is that all the "versions" (flat, sharp, etc.) of any particular note are very far-flung. D♭♭ and D𝄪 are almost a fifth apart, and don't feel like different versions of the same note. The three black keys between A and B, which do feel like different versions of the same note, can be notated as some version of G, A, B, C or D. The connection between a note in all its versions and an approximate pitch range is broken.

Another approach to naming 22-edo keys is to keep this connection by ensuring that all versions of a note fall between the neighboring white keys. For example, all versions of C are contained between the B and the D keys. Each black key's name is some version of one of the two nearest white keys. For example, the black keys between C and D are only named as some version of C or D, never as some version of B or E. A white key always has the obvious name. Unfortunately in some edos, occasional "out of bounds" notes are unavoidable. Even 12-edo has F♯ major with an E♯ note, or G♭ major with a C♭. Such notes occur in the tables for 22-edo and 41-edo that follow.

To achieve this, ups and downs are allowed in tonic names and key signatures. **If the tonic has an up or down, all seven notes in the scale do as well.** Avoid naming notes as double-sharps or double-flats. Also avoid E♯, F♭, etc.

A major scale starting on the C♯ key wouldn't be C♯ major, because that would contain E♯. Instead it is ⩛D major. There is much less overlap between major key names and minor key names. Alternate keys are in parentheses:

Table 4.7 – Preferred tonic names and key signatures for 22-edo, using ups and downs

major keys	C	D♭ (^C)	^D♭	vD	D	E♭	^E♭	vE	E	F	^F	vF♯ ^G♭	vG	G	A♭	^A♭	vA	A	B♭	^B ♭	vB	B (vC)
minor keys	"	^C	vC♯	C♯	"	^D	vD♯ ^E♭	"	"	"	"	vF♯	F♯	"	^G	vG♯	G♯ (vA)	"	B♭ (^A)	"	"	B

Major keys are mostly natural, down, upflat or flat. Likewise, minor keys are mostly natural, up, downsharp or sharp.

An ordinary modulation by a fifth from ^F major to D♭ major would look very odd on paper. In this case, for D♭ major one might use an alternate key, ^C major. Likewise, one might use vC major, ^A minor, or vA minor.

Some key signatures contain a "global" up or down that raises or lowers all seven notes, written on the staff as a circled ^ or v, and written in the table below as (^) and (v). The first and last rows are the same notes in 22-edo, ^G♭ = vF♯.

Table 4.8 – 22-edo key signatures using ups and downs, in chain-of-5ths order ("out-of-bounds" names are underlined)

key signature	major key	major scale	minor key	minor scale
♭ ♭ ♭ ♭ ♭ ♭ (^)	^G♭ major	^G♭ ^A♭ ^B♭ **^C♭** ^D♭ ^E♭ ^F ^G♭	^E♭ minor	^E♭ ^F ^G♭ ^A♭ ^B♭ **^C♭** ^D♭ ^E♭
♭ ♭ ♭ ♭ ♭ (^)	^D♭ major	^D♭ ^E♭ ^F ^G♭ ^A♭ ^B♭ ^C ^D♭	^B♭ minor	^B♭ ^C ^D♭ ^E♭ ^F ^G♭ ^A♭ ^B♭
♭ ♭ ♭ ♭ (^)	^A♭ major	^A♭ ^B♭ ^C ^D♭ ^E♭ ^F ^G ^A♭	^F minor	^F ^G ^A♭ ^B♭ ^C ^D♭ ^E♭ ^F
♭ ♭ ♭ (^)	^E♭ major	^E♭ ^F ^G ^A♭ ^B♭ ^C ^D ^E♭	^C minor	^C ^D ^E♭ ^F ^G ^A♭ ^B♭ ^C
♭ ♭ (^)	^B♭ major	^B♭ ^C ^D ^E♭ ^F ^G ^A ^B♭	^G minor	^G ^A ^B♭ ^C ^D ^E♭ ^F ^G
♭ (^)	^F major	^F ^G ^A ^B♭ ^C ^D ^E ^F	^D minor	^D ^E ^F ^G ^A ^B♭ ^C ^D
(^) ♭ ♭ ♭ ♭ ♭	(^C major) D♭ major	(^C ^D ^E ^F ^G ^A ^B ^C) D♭ E♭ F G♭ A♭ B♭ C D♭	(^A minor) B♭ minor	(^A ^B ^C ^D ^E ^F ^G ^A) B♭ C D♭ E♭ F G♭ A♭ B♭
♭ ♭ ♭ ♭	A♭ major	A♭ B♭ C D♭ E♭ F G A♭	F minor	F G A♭ B♭ C D♭ E♭ F
♭ ♭ ♭	E♭ major	E♭ F G A♭ B♭ C D E♭	C minor	C D E♭ F G A♭ B♭ C
♭ ♭	B♭ major	B♭ C D E♭ F G A B♭	G minor	G A B♭ C D E♭ F G
♭	F major	F G A B♭ C D E F	D minor	D E F G A B♭ C D
no sharps or flats	C major	C D E F G A B C	A minor	A B C D E F G A
♯	G major	G A B C D E F♯ G	E minor	E F♯ G A B C D E
♯ ♯	D major	D E F♯ G A B C♯ D	B minor	B C♯ D E F♯ G A B
♯ ♯ ♯	A major	A B C♯ D E F♯ G♯ A	F♯ minor	F♯ G♯ A B C♯ D E F♯
♯ ♯ ♯ ♯	E major	E F♯ G♯ A B C♯ D♯ E	C♯ minor	C♯ D♯ E F♯ G♯ A B C♯
♯ ♯ ♯ ♯ ♯ (v)	B major (vC major)	B C♯ D♯ E F♯ G♯ A♯ B (vC vD vE vF vG vA vB vC)	G♯ minor (vA minor)	G♯ A♯ B C♯ D♯ E♯ F G♯ (vA vB vC vD vE vF vG vA)
♯ (v)	vG major	vG vA vB vC vD vE vF♯ vG	vE minor	vE vF♯ vG vA vB vC vD vE
♯ ♯ (v)	vD major	vD vE vF♯ vG vA vB vC♯ vD	vB minor	vB vC♯ vD vE vF♯ vG vA vB
♯ ♯ ♯ (v)	vA major	vA vB vC♯ vD vE vF♯ vG♯ vA	vF♯ minor	vF♯ vG♯ vA vB vC♯ vD vE vF♯
♯ ♯ ♯ ♯ (v)	vE major	vE vF♯ vG♯ vA vB vC♯ vD♯ vE	vC♯ minor	vC♯ vD♯ vE vF♯ vG♯ vA vB vC♯
♯ ♯ ♯ ♯ ♯ (v)	vB major	vB vC♯ vD♯ vE vF♯ vG♯ vA♯ vB	vG♯ minor	vG♯ vA♯ vB vC♯ vD♯ vE♯ vF vG♯
♯ ♯ ♯ ♯ ♯ ♯ (v)	vF♯ major	vF♯ vG♯ vA♯ vB vC♯ vD♯ **vE♯** vF♯	vD♯ minor	vD♯ **vE♯** vF♯ vG♯ vA♯ vB vC♯ vD♯

Table 4.9 – 22-edo key signatures using ups and downs, in melodic order ("out-of-bounds" names are underlined)

major key	major scale	key signature	minor key	minor scale
C major	C D E F G A B C	no sharps or flats	A minor	A B C D E F G A
D♭ major (^C major)	D♭ E♭ F G♭ A♭ B♭ C D♭ (^C ^D ^E ^F ^G ^A ^B ^C)	♭ ♭ ♭ ♭ ♭ (^)	B♭ minor (^A minor)	B♭ C D♭ E♭ F G♭ A♭ B♭ (^A ^B ^C ^D ^E ^F ^G ^A)
^D♭ major	^D♭ ^E♭ ^F ^G♭ ^A♭ ^B♭ ^C ^D♭	♭ ♭ ♭ ♭ ♭ (^)	^B♭ minor	^B♭ ^C ^D♭ ^E♭ ^F ^G♭ ^A♭ ^B♭
vD major	vD vE vF♯ vG vA vB vC♯ vD	♯ ♯ (v)	vB minor	vB vC♯ vD vE vF♯ vG vA vB
D major	D E F♯ G A B C♯ D	♯ ♯	B minor	B C♯ D E F♯ G A B
E♭ major	E♭ F G A♭ B♭ C D E♭	♭ ♭ ♭	C minor	C D E♭ F G A♭ B♭ C
^E♭ major	^E♭ ^F ^G ^A♭ ^B♭ ^C ^D ^E♭	♭ ♭ ♭ (^)	^C minor	^C ^D ^E♭ ^F ^G ^A♭ ^B♭ ^C
vE major	vE vF♯ vG♯ vA vB vC♯ vD♯ vE	♯ ♯ ♯ ♯ (v)	vC♯ minor	vC♯ vD♯ vE vF♯ vG♯ vA vB vC♯
E major	E F♯ G♯ A B C♯ D♯ E	♯ ♯ ♯ ♯	C♯ minor	C♯ D♯ E F♯ G♯ A B C♯
F major	F G A B♭ C D E F	♭	D minor	D E F G A B♭ C D
^F major	^F ^G ^A ^B♭ ^C ^D ^E ^F	♭ (^)	^D minor	^D ^E ^F ^G ^A ^B♭ ^C ^D
vF♯ major ^G♭ major	vF♯ vG♯ vA♯ vB vC♯ vD♯ **vE♯** vF♯ ^G♭ ^A♭ ^B♭ **^C♭** ^D♭ ^E♭ ^F ^G♭	♯ ♯ ♯ ♯ ♯ ♯ (v) ♭ ♭ ♭ ♭ ♭ ♭ (^)	vD♯ minor ^E♭ minor	vD♯ **vE♯** vF♯ vG♯ vA♯ vB vC♯ vD♯ ^E♭ ^F ^G♭ ^A♭ ^B♭ **^C♭** ^D♭ ^E♭
vG major	vG vA vB vC vD vE vF♯ vG	♯ (v)	vE minor	vE vF♯ vG vA vB vC vD vE
G major	G A B C D E F♯ G	♯	E minor	E F♯ G A B C D E
A♭ major	A♭ B♭ C D♭ E♭ F G A♭	♭ ♭ ♭ ♭	F minor	F G A♭ B♭ C D♭ E♭ F
^A♭ major	^A♭ ^B♭ ^C ^D♭ ^E♭ ^F ^G ^A♭	♭ ♭ ♭ ♭ (^)	^F minor	^F ^G ^A♭ ^B♭ ^C ^D♭ ^E♭ ^F
vA major	vA vB vC♯ vD vE vF♯ vG♯ vA	♯ ♯ ♯ (v)	vF♯ minor	vF♯ vG♯ vA vB vC♯ vD vE vF♯
A major	A B C♯ D E F♯ G♯ A	♯ ♯ ♯	F♯ minor	F♯ G♯ A B C♯ D E F♯
B♭ major	B♭ C D E♭ F G A B♭	♭ ♭	G minor	G A B♭ C D E♭ F G
^B♭ major	^B♭ ^C ^D ^E♭ ^F ^G ^A ^B♭	♭ ♭ (^)	^G minor	^G ^A ^B♭ ^C ^D ^E♭ ^F ^G
vB major	vB vC♯ vD♯ vE vF♯ vG♯ vA♯ vB	♯ ♯ ♯ ♯ ♯ (v)	vG♯ minor	vG♯ vA♯ vB vC♯ vD♯ vE♯ vF vG♯
B major (vC major)	B C♯ D♯ E F♯ G♯ A♯ B (vC vD vE vF vG vA vB vC)	♯ ♯ ♯ ♯ ♯ (v)	G♯ minor (vA minor)	G♯ A♯ B C♯ D♯ E♯ F G♯ (vA vB vC vD vE vF vG vA)

Both methods of assigning key signatures have their advantages. A quick comparison:

Table 4.10 – Preferred tonic names and key signatures for 22-edo

steps	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
cents	0¢	55¢	109	164	218	273	327	382	436	491	545	600	655	709	764	818	873	927	982	1036	1091	1145

using ^v	P1	m2	^m2	vM2	M2	m3	^m3	vM3	M3	P4	^4 d5	vA4 ^d5	A4 v5	P5	m6	^m6	vM6	M6	m7	^m7	vM7	M7
major keys	C	D♭ (^C)	^D♭	vD	D	E♭	^E♭	vE	E	F	^F	vF♯ ^G♭	vG	G	A♭	^A♭	vA	A	B♭	^B♭	vB	B (vC)
minor keys	"	^C	vC♯	C♯	"	^D	vD♯ ^E♭	"	"	"	"	vF♯	F♯	"	^G	vG♯	G♯ (vA)	"	B♭ (^A)	"	"	B

no ^v	P1	m2	d3	A1	M2	m3	d4	A2	M3	P4	d5	A3	A4	P5	m6	d7	A5	M6	m7	d8	A6	M7
major	C	D♭	E♭♭	C♯	D	E♭	F♭	D♯	E	F	G♭	E♯	F♯	G	A♭	B♭♭	G♯	A	B♭	C♭	A♯	B
minor	"	"	B♯	"	"	"	"	"	"	"	"	"	"	"	"	F𝄫	"	"	"	"	"	"

22-edo's major scale sounds better lowering the third, sixth and seventh. One might be tempted to use key signatures such as ♯ v♯ v♯ v♯ for E downmajor = E F♯ vG♯ A B vC♯ vD♯ E. But this scale has a wolf fifth from F♯ to vC♯. Every time the unaltered F and C appear together on the score, it looks like a perfect fifth but doesn't sound like one.

Furthermore, the key signature's main function is to indicate the key, and a general sense of major vs. minor, but not the exact scale. Here's why: the key signature is meant to be not only readable but speed-readable. The fewer possible key signatures there are, the more instantly recognizable they are. Packing too much information into the key signature inhibits rapid sight reading. We're already asking the musician to cope with new elements in key signatures: for the first method, either double sharps or double flats, and for the second method, either a global up or a global down. That's asking a lot, so it's better to use only the standard key signatures.

However, if desired, analogous to a color signature, one can use an ups/downs signature reading vC♯ vD♯ vG♯. This would be placed immediately above the key signature on every line.

The first page of "Tibia" is written out below in the key of vF♯. This is a rather extreme example, a very chromatic yet very tonal song in a very remote key. There are many dups and duds. The first measure has vvA♯, not ^A, because the vF♯ chord has a vM3, not a ^^m3. This is analogous to spelling an A♯ major chord with a C𝄪, not a D. In measures 6 and 8, this is taken further, and dud double-sharps are used instead of upsharps.

This example uses independent ups and downs. It uses the bare minimum of accidentals needed. In practice, there would be many courtesy accidentals. Here are all the notes used. The top staff follows the P1 – ^1 – vA1 – A1 – M2 – ^M2 – vA2 – A2 – M3 – P4pattern of Figure 4.1, and the bottom staff follows the P1 – m2 – ^m2 – vM2 – M2 – m3 – ^m3 – vM3 – M3 – P4 pattern.

Figure 4.6 – 22-edo staff notation in vF♯ with mandatory accidentals

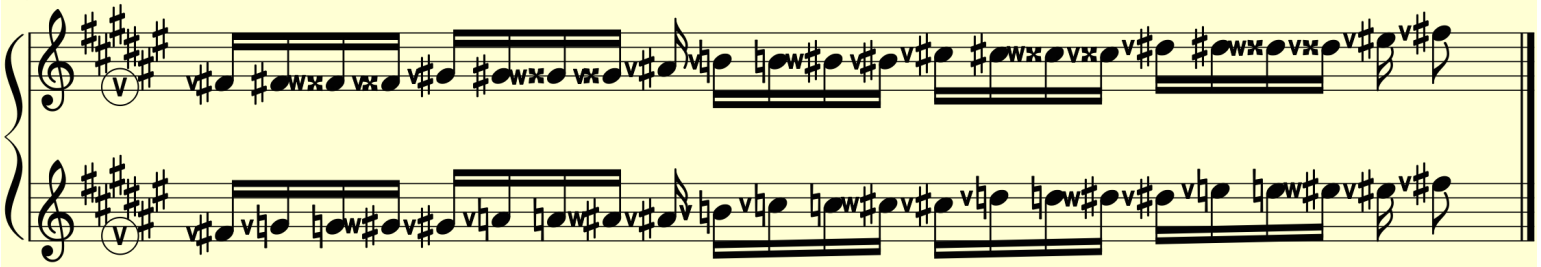
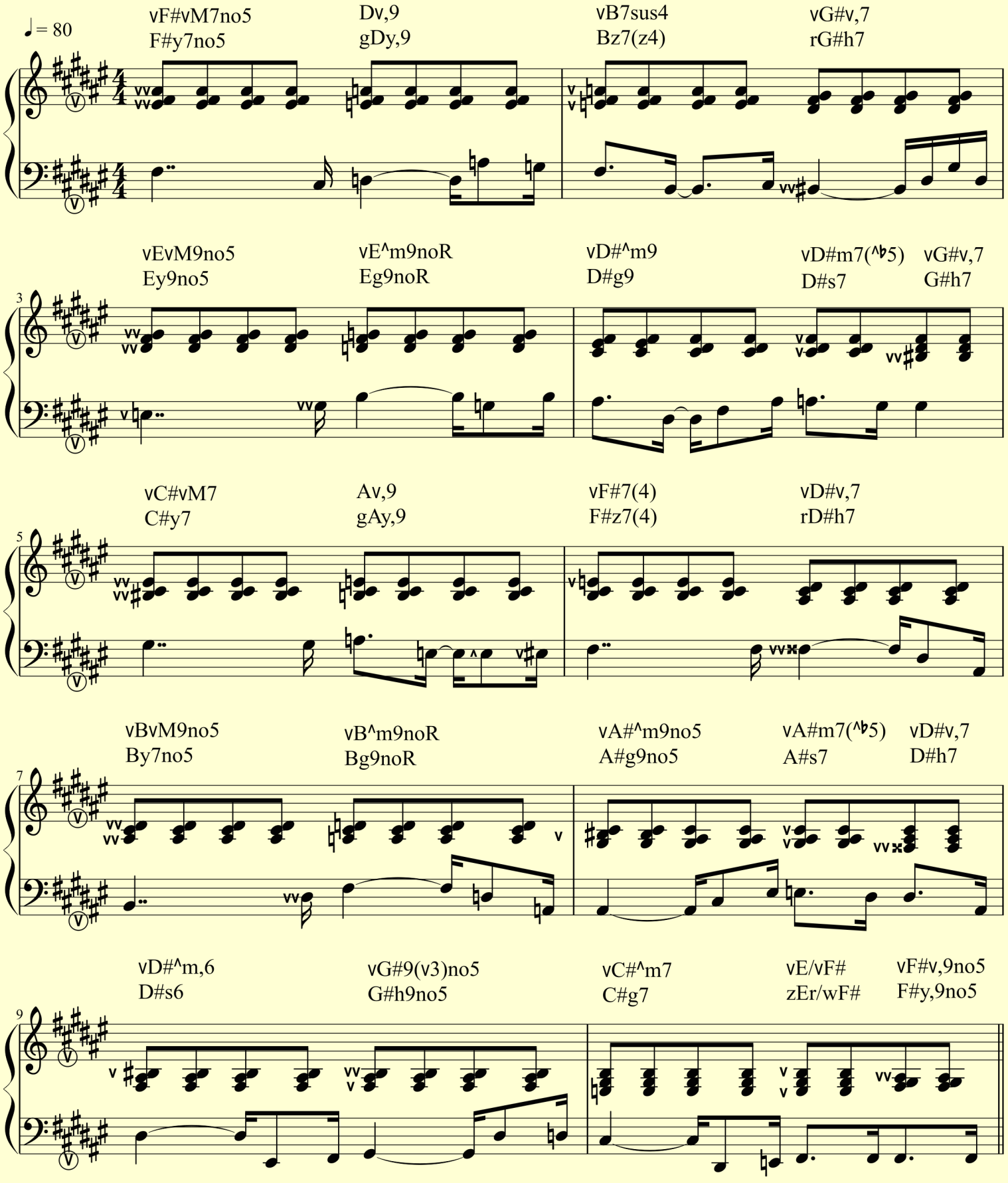


Figure 4.7 – "Tibia" in vF♯ with independent ups and downs

Tibia in vF♯ in 22edo

Paul Erlich

Ⓥ makes all seven notes default to down



17-edo is notated much like 22-edo, and the 22-edo section explains what issues arise.

Table 4.11 – 17-edo notation, with preferred tonic names and key signatures

steps	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
edo cents	0¢	71	141	212	282	353	424	494	565	635	706	776	847	918	988	1059	1129	1200
intervals (with ^v)	P1	^1 m2	A1 ~2	M2	m3	~3 d4	M3 v4	P4	^4 d5	A4 v5	P5	^5 m6	A5 ~6	M6	m7	~7 d8	M7 v8	P8
major keys	**D**	E♭	^E♭ (vE)	**E**	**F**	^F	vG	**G**	A♭	vA (^A♭)	**A**	B♭	vB	**B** (vC)	**C**	D♭ (^C)	vD	**D**
minor keys	"	^D	vE	"	"	vF♯ (^F)	F♯	"	^G	G♯ (vA)	"	B♭ (^A)	"	**B**	"	^C (vC♯)	C♯	"
intervals (w/out ^v)	P1	m2	A1 d3	M2	m3	A2 d4	M3	P4	d5	A4	P5	m6	A5 d7	M6	m7	d8	M7	P8
major keys	**D**	E♭	F♭ (D♯)	**E**	**F**	G♭	F♯	**G**	A♭	G♯	**A**	B♭	C♭	**B**	**C**	D♭	C♯	**D**
minor keys	"	"	D♯	"	"	E♯ (G♭)	"	"	"	"	"	"	A♯	"	"	"	"	"

Table 4.12 – 17-edo key signatures using ups and downs, in chain-of-fifths order

key signature	major key	major scale	minor key	minor scale
♭ ♭ ♭ ♭ (^)	(^A♭ major)	(^A♭ ^B♭ ^C ^D♭ ^E♭ ^F ^G ^A♭)	(^F minor)	(^F ^G ^A♭ ^B♭ ^C ^D♭ ^E♭ ^F)
♭ ♭ ♭ (^)	^E♭ major	^E♭ ^F ^G ^A♭ ^B♭ ^C ^D ^E♭	^C minor	^C ^D ^E♭ ^F ^G ^A♭ ^B♭ ^C
♭ ♭ (^)	^B♭ major	^B♭ ^C ^D ^E♭ ^F ^G ^A ^B♭	^G minor	^G ^A ^B♭ ^C ^D ^E♭ ^F ^G
♭ (^)	^F major	^F ^G ^A ^B♭ ^C ^D ^E ^F	^D minor	^D ^E ^F ^G ^A ^B♭ ^C ^D
(^) ♭ ♭ ♭ ♭ ♭	(^C major) D♭ major	(^C ^D ^E ^F ^G ^A ^B ^C) D♭ E♭ F G♭ A♭ B♭ C D♭	(^A minor) B♭ minor	(^A ^B ^C ^D ^E ^F ^G ^A) B♭ C D♭ E♭ F G♭ A♭ B♭
♭ ♭ ♭ ♭	A♭ major	A♭ B♭ C D♭ E♭ F G A♭	F minor	F G A♭ B♭ C D♭ E♭ F
♭ ♭ ♭	E♭ major	E♭ F G A♭ B♭ C D E♭	C minor	C D E♭ F G A♭ B♭ C
♭ ♭	B♭ major	B♭ C D E♭ F G A B♭	G minor	G A B♭ C D E♭ F G
♭	F major	F G A B♭ C D E F	D minor	D E F G A B♭ C D
no sharps or flats	C major	C D E F G A B C	A minor	A B C D E F G A
♯	G major	G A B C D E F♯ G	E minor	E F♯ G A B C D E
♯ ♯	D major	D E F♯ G A B C♯ D	B minor	B C♯ D E F♯ G A B
♯ ♯ ♯	A major	A B C♯ D E F♯ G♯ A	F♯ minor	F♯ G♯ A B C♯ D E F♯
♯ ♯ ♯ ♯	E major	E F♯ G♯ A B C♯ D♯ E	C♯ minor	C♯ D♯ E F♯ G♯ A B C♯
♯ ♯ ♯ ♯ ♯ (v)	B major (vC major)	B C♯ D♯ E F♯ G♯ A♯ B (vC vD vE vF vG vA vB vC)	G♯ minor (vA minor)	G♯ A♯ B C♯ D♯ E♯ F G♯ (vA vB vC vD vE vF vG vA)
♯ (v)	vG major	vG vA vB vC vD vE vF♯ vG	vE minor	vE vF♯ vG vA vB vC vD vE
♯ ♯ (v)	vD major	vD vE vF♯ vG vA vB vC♯ vD	vB minor	vB vC♯ vD vE vF♯ vG vA vB
♯ ♯ ♯ (v)	vA major	vA vB vC♯ vD vE vF♯ vG♯ vA	vF♯ minor	vF♯ vG♯ vA vB vC♯ vD vE vF♯
♯ ♯ ♯ ♯ (v)	(vE major)	(vE vF♯ vG♯ vA vB vC♯ vD♯ vE)	(vC♯ minor)	(vC♯ vD♯ vE vF♯ vG♯ vA vB vC♯)

Table 4.13 – 17-edo key signatures using ups and downs, in melodic order

key signature	major key	major scale	minor key	minor scale
no sharps or flats	C major	C D E F G A B C	A minor	A B C D E F G A
♭ ♭ ♭ ♭ ♭ (^)	D♭ major (^C major)	D♭ E♭ F G♭ A♭ B♭ C D♭ (^C ^D ^E ^F ^G ^A ^B ^C)	B♭ minor (^A minor)	B♭ C D♭ E♭ F G♭ A♭ B♭ (^A ^B ^C ^D ^E ^F ^G ^A)
♯ ♯ (v)	vD major	vD vE vF♯ vG vA vB vC♯ vD	vB minor	vB vC♯ vD vE vF♯ vG vA vB
♯ ♯	D major	D E F♯ G A B C♯ D	B minor	B C♯ D E F♯ G A B
♭ ♭ ♭	E♭ major	E♭ F G A♭ B♭ C D E♭	C minor	C D E♭ F G A♭ B♭ C
♭ ♭ ♭ (^) ♯ ♯ ♯ ♯ (v)	^E♭ major (vE major)	^E♭ ^F ^G ^A♭ ^B♭ ^C ^D ^E♭ (vE vF♯ vG♯ vA vB vC♯ vD♯ vE)	^C minor (vC♯ minor)	^C ^D ^E♭ ^F ^G ^A♭ ^B♭ ^C (vC♯ vD♯ vE vF♯ vG♯ vA vB vC♯)
♯ ♯ ♯ ♯	E major	E F♯ G♯ A B C♯ D♯ E	C♯ minor	C♯ D♯ E F♯ G♯ A B C♯
♭	F major	F G A B♭ C D E F	D minor	D E F G A B♭ C D
♭ (^)	^F major	^F ^G ^A ^B♭ ^C ^D ^E ^F	^D minor	^D ^E ^F ^G ^A ^B♭ ^C ^D
♯ (v)	vG major	vG vA vB vC vD vE vF♯ vG	vE minor	vE vF♯ vG vA vB vC vD vE
♯	G major	G A B C D E F♯ G	E minor	E F♯ G A B C D E
♭ ♭ ♭ ♭	A♭ major	A♭ B♭ C D♭ E♭ F G A♭	F minor	F G A♭ B♭ C D♭ E♭ F
♯ ♯ ♯ (v) ♭ ♭ ♭ ♭ (^)	vA major (^A♭ major)	vA vB vC♯ vD vE vF♯ vG♯ vA (^A♭ ^B♭ ^C ^D♭ ^E♭ ^F ^G ^A♭)	vF♯ minor (^F minor)	vF♯ vG♯ vA vB vC♯ vD vE vF♯ (^F ^G ^A♭ ^B♭ ^C ^D♭ ^E♭ ^F)
♯ ♯ ♯	A major	A B C♯ D E F♯ G♯ A	F♯ minor	F♯ G♯ A B C♯ D E F♯
♭ ♭	B♭ major	B♭ C D E♭ F G A B♭	G minor	G A B♭ C D E♭ F G
♭ ♭ (^)	^B♭ major	^B♭ ^C ^D ^E♭ ^F ^G ^A ^B♭	^G minor	^G ^A ^B♭ ^C ^D ^E♭ ^F ^G
♯ ♯ ♯ ♯ ♯ (v)	B major (vC major)	B C♯ D♯ E F♯ G♯ A♯ B (vC vD vE vF vG vA vB vC)	G♯ minor (vA minor)	G♯ A♯ B C♯ D♯ E♯ F G♯ (vA vB vC vD vE vF vG vA)

24-edo: The 24-edo circle of fifths closes before it reaches all the notes, thus it takes two circles or **rings** to name all the notes. Ups and downs are used to distinguish between the different rings. 24-edo has a plain (neither up nor down) ring and an up ring. All the up notes can alternatively be written as down notes.

plain ring: E♭ – B♭ – F – C – G – D – A – E – B – F♯ – C♯ – G♯/A♭ – E♭

up ring: ^E♭ – ^B♭ – ^F – ^C – ^G – ^D – ^A – ^E – ^B – ^F♯ – ^C♯ – ^G♯/^A♭ – ^E♭

17, 19 and 22 are **single-ring** edos and 24 is a **multi-ring** edo. On the scale tree, multi-ring edos are on the vertical dotted lines. The further down the dotted line, the more rings there are.

Multi-ring edos use ups and downs for a different reason than single-ring edos do, and they absolutely require ups and downs. In a single-ring edo, we can require that the tonic be a plain note. But multi-ring edos force the use of tonics that are not plain.

24-edo key signatures are like conventional ones, but with the possible addition of the (^) or (v) symbol that raises or lowers all notes of the scale. A few keys can be written two different ways. ^E major with ♯ ♯ ♯ ♯ (^) could instead be vF major with ♭ (v).

Table 4.14 – 24-edo notation

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
0¢	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800	850	900	950	1000	1050	1100	1150	
P1	^1 vm2	A1 m2	^A1 ~2	M2	^M2 vm3	A2 m3	~3	M3 d4	^M3 v4	P4	^4 vd5	A4 d5	^A4 v5	P5	^5 vm6	A5 m6	~6	M6 d7	^M6 vm7	m7	~7 vd8	M7 d8	^M 7 v8	P8
D	^D vE♭	D♯ E♭	^D♯ vE	**E**	^E vF	**F**	^F vG♭	F♯ G♭	^F♯ vG	**G**	^G vA♭	G♯ A♭	^G♯ vA	**A**	^A vB♭	A♯ B♭	^A♯ vB	**B**	^B vC	**C**	^C vD♭	C♯ D♭	^C♯ vD	**D**

Table 4.15 – Preferred tonic names and key signatures for 24-edo

major keys	**D**	^D	E♭	vE	**E**	^E vF	**F**	^F	F♯ G♭	vG	**G**	^G	A♭	vA	**A**	^A	B♭	vB	**B**	^B vC	**C**	^C	D♭	vD
minor keys	"	"	D♯ E♭	"	"	"	"	"	F♯	"	"	"	G♯	"	"	"	"	"	"	"	"	C♯	"	

26-edo is notated much like 19-edo. There can be up to six double-sharps or double-flats in the key signature.

Table 4.16 – 26-edo notation

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
0¢	46	92	138	185	231	277	323	369	415	462	508	554	600	646	692	738	785	831	877	923	969	1015	1062	1108	1154	1200
P1	A1	d2	m2	M2	A2	d3	m3	M3	A3	d4	P4	A4	AA4 dd5	d5	P5	A5	d6	m6	M6	A6	d7	m7	M7	A7	d8	P8
D	D♯	E♭♭	E♭	**E**	E♯	F♭	**F**	F♯	F^{X}	G♭	**G**	G♯	G^{X} A♭♭	A♭	**A**	A♯	B♭♭	B♭	**B**	B♯	C♭	**C**	C♯	C^{X}	D♭	**D**

Table 4.17 – Preferred tonic names and key signatures for 26-edo

major keys	**D**	D♯	E♭♭	E♭	**E**	E♯	F♭	**F**	F♯	F^{X} G♭♭	G♭	**G**	G♯	A♭♭	A♭	**A**	A♯	B♭♭	B♭	**B**	B♯	C♭	**C**	C♯	D♭♭	D♭
minor keys	"	"	D^{X} E♭♭	"	"	"	"	"	"	F^{X}	"	"	"	G^{X}	"	"	"	"	"	"	"	"	"	"	C^{X}	"

27-edo is notated much like 22-edo, and the 22-edo section explains what issues arise.

Table 4.18 – 27-edo notation (* asterisk indicates keys with out-of-bounds notes)

steps	edo cents	interval (with ^v)	major keys	minor keys	interval (no ^v)	major keys	minor keys
0	0¢	P1	**D**	"	P1	**D**	"
1	44	^1 / m2	E♭	^D	m2	E♭	"
2	89	^m2	^E♭	" *	d3	F♭	"
3	133	~2	vvE	" *	dd4 / -d2	G♭♭ (C^X)	"
4	178	vM2	vE	"	A1	D♯	"
5	222	M2	**E**	"	M2	**E**	"
6	267	m3	**F**	"	m3	**F**	"
7	311	^m3	^F	"	d4	G♭	"
8	356	~3	^^F	"	dd5 / AA1	A♭♭	D^X (A♭♭)
9	400	vM3	vF♯ *		A2	E♯	"
10	444	M3 / v4	F♯ *		M3	F♯	"
11	489	P4	**G**	"	P4	**G**	"
12	533	^4 / d5	A♭	^G	d5	A♭	"
13	578	^^4 / ^d5	^^G		d6	B♭♭	"
14	622	vA4 / vv5	vvA		A3 / dd7	F^X (C♭♭)	"
15	667	A4 / v5	vA		A4	G♯	"
16	711	P5	**A**	"	P5	**A**	"
17	756	^5 / m6	B♭		m6	B♭	"
18	800	^m6	^B♭		d7	C♭	"
19	844	~6	vvB		dd8 / AA4	D♭♭	G^X
20	889	vM6	vB		A5	A♯	"
21	933	M6	**B**	"	M6	**B**	"
22	978	m7	**C**	"	m7	**C**	"
23	1022	^m7	D♭	^C	d8	D♭	"
24	1067	~7	^^C		d9 / AA5	E♭♭	E♭♭ (A^X)
25	1111	vM7	vC♯ *		A6	B♯	"
26	1156	M7	vD	C♯	M7	C♯	"
27	1200	P8	**D**	"	P8	**D**	"

31-edo: If using plain tonics, 2 major keys and 2 minor keys require triple-sharps and triple-flats. It's probably better to use upped and downed tonics.

Table 4.19 – 31-edo notation (* asterisks indicate 4 keys that require triple-sharps or triple-flats)

steps	edo cents	interval (with ^v)	major keys	minor keys	interval (no ^v)	major keys	minor keys
0	0¢	P1	**D**	"	P1	**D**	"
1	39	^1	^D	"	d2	E𝄫	"
2	77	A1 / vm2	vE♭	D♯	A1	D♯	"
3	116	m2	E♭	"	m2	E♭	"
4	155	~2	^E♭ / vE	vE	AA1 / dd3	F𝄫 *	D^{X}
5	194	M2	**E**	"	M2	**E**	"
6	232	^M2	^E	"	d3	F♭	"
7	271	vm3	vF	"	A2	E♯	"
8	310	m3	**F**	"	m3	**F**	"
9	348	~3	^F	vF♯ / ^F	AA2 / dd4	G𝄫	E^{X} *
10	387	M3	F♯	"	M3	F♯	"
11	426	^M3 / d4	G♭	^F♯	d4	G♭	"
12	465	v4	vG	"	A3	F^{X}	"
13	503	P4	**G**	"	P4	**G**	"
14	542	^4	^G	"	AA3 / dd5	A𝄫	"
15	581	A4 / vd5	vA♭	G♯	A4	G♯	"
16	619	d5	A♭	A♭ / ^G♯	d5	A♭	"
17	658	v5	vA / ^A♭	vA	AA4 / dd6	G^{X} *	"
18	697	P5	**A**	"	P5	**A**	"
19	735	^5	^A	"	d6	B𝄫	"
20	774	A5 / vm6	vB♭	A♯ / vB♭	A5	A♯	"
21	813	m6	B♭	"	m6	B♭	"
22	852	~6	^B♭	vB	AA5 / dd7	C𝄫	A^{X}
23	890	M6	**B**	"	M6	**B**	"
24	929	^M6	^B / C♭	^B	d7	C♭	"
25	968	vm7	vC	"	A6	B♯	"
26	1006	m7	**C**	"	m7	**C**	"
27	1045	~7	^C	^C / vC♯	AA6 / dd8	D𝄫	" *
28	1084	M7	C♯ / vD♭	C♯	M7	C♯	"
29	1123	^M7 / d8	D♭	^C♯	d8	D♭	"
30	1161	v8	vD	"	A7	C^{X}	"
31	1200	P8	**D**	"	P8	**D**	"

Table 4.20 – 31-edo key signatures using ups and downs, in chain-of-fifths order

key signature	major key	major scale	minor key	minor scale
♭ ♭ ♭ ♭ (^)	(^A♭ major)	(^A♭ ^B♭ ^C ^D♭ ^E♭ ^F ^G ^A♭)	(^F minor)	(^F ^G ^A♭ ^B♭ ^C ^D♭ ^E♭ ^F)
♭ ♭ ♭ (^)	^E♭ major	^E♭ ^F ^G ^A♭ ^B♭ ^C ^D ^E♭	^C minor	^C ^D ^E♭ ^F ^G ^A♭ ^B♭ ^C
♭ ♭ (^)	^B♭ major	^B♭ ^C ^D ^E♭ ^F ^G ^A ^B♭	^G minor	^G ^A ^B♭ ^C ^D ^E♭ ^F ^G
♭ (^)	^F major	^F ^G ^A ^B♭ ^C ^D ^E ^F	^D minor	^D ^E ^F ^G ^A ^B♭ ^C ^D
(^)	^C major	^C ^D ^E ^F ^G ^A ^B ^C	^A minor	^A ^B ^C ^D ^E ^F ^G ^A
♯ (^)	^G major	^G ^A ^B ^C ^D ^E ^F♯ ^G	^E minor	^E ^F♯ ^G ^A ^B ^C ^D ^E
♯ ♯ (^)	^D major	^D ^E ^F♯ ^G ^A ^B ^C♯ ^D	^B minor	^B ^C♯ ^D ^E ^F♯ ^G ^A ^B
♯ ♯ ♯ (^)	^A major	^A ^B ^C♯ ^D ^E ^F♯ ^G♯ ^A	^F♯ minor	^F♯ ^G♯ ^A ^B ^C♯ ^D ^E ^F♯
♯ ♯ ♯ ♯ (^)	^E major	^E ^F♯ ^G♯^A ^B ^C♯ ^D♯ ^E	^C♯ minor	^C♯ ^D♯ ^E ^F♯ ^G♯ ^A ^B ^C♯
♯ ♯ ♯ ♯ ♯ (^) ♭ ♭ ♭ ♭ ♭ ♭ ♭	^B major C♭ major	^B ^C♯ ^D♯ ^E ^F♯ ^G♯ ^A♯ ^B C♭ D♭ E♭ F♭ G♭ A♭ B♭ C♭	^G♯ minor A♭ minor	^G♯ ^A♯ ^B ^C♯ ^D♯ ^E♯ ^F ^G♯ A♭ B♭ C♭ D♭ E♭ F♭ G♭ A♭
♭ ♭ ♭ ♭ ♭ ♭	G♭ major	G♭ A♭ B♭ C♭ D♭ E♭ F G♭	E♭ minor	E♭ F G♭ A♭ B♭ C♭ D♭ E♭
♭ ♭ ♭ ♭ ♭	D♭ major	D♭ E♭ F G♭ A♭ B♭ C D♭	B♭ minor	B♭ C D♭ E♭ F G♭ A♭ B♭
♭ ♭ ♭ ♭	A♭ major	A♭ B♭ C D♭ E♭ F G A♭	F minor	F G A♭ B♭ C D♭ E♭ F
♭ ♭ ♭	E♭ major	E♭ F G A♭ B♭ C D E♭	C minor	C D E♭ F G A♭ B♭ C
♭ ♭	B♭ major	B♭ C D E♭ F G A B♭	G minor	G A B♭ C D E♭ F G
♭	F major	F G A B♭ C D E F	D minor	D E F G A B♭ C D
no sharps or flats	C major	C D E F G A B C	A minor	A B C D E F G A
♯	G major	G A B C D E F♯ G	E minor	E F♯ G A B C D E
♯ ♯	D major	D E F♯ G A B C♯ D	B minor	B C♯ D E F♯ G A B
♯ ♯ ♯	A major	A B C♯ D E F♯ G♯ A	F♯ minor	F♯ G♯ A B C♯ D E F♯
♯ ♯ ♯ ♯	E major	E F♯ G♯ A B C♯ D♯ E	C♯ minor	C♯ D♯ E F♯ G♯ A B C♯
♯ ♯ ♯ ♯ ♯	B major	B C♯ D♯ E F♯ G♯ A♯ B	G♯ minor	G♯ A♯ B C♯ D♯ E♯ F G♯
♯ ♯ ♯ ♯ ♯ ♯	F♯ major	F♯ G♯ A♯ B C♯ D♯ E♯ F♯	D♯ minor	D♯ E♯ F♯ G♯ A♯ B C♯ D♯
♯ ♯ ♯ ♯ ♯ ♯ ♯ ♭ ♭ ♭ ♭ ♭ (v)	C♯ major vD♭ major	C♯ D♯ E♯ F♯ G♯ A♯ B♯ C♯ vD♭ vE♭ vF vG♭ vA♭ vB♭ vC vD♭	A♯ minor vB♭ minor	A♯ B♯ C♯ D♯ E♯ F♯ G♯ A♯ vB♭ vC vD♭ vE♭ vF vG♭ vA♭ vB♭
♭ ♭ ♭ ♭ (v)	vA♭ major	vA♭ vB♭ vC vD♭ vE♭ vF vG vA♭	vF minor	vF vG vA♭ vB♭ vC vD♭ vE♭ F
♭ ♭ ♭ (v)	vE♭ major	vE♭ vF vG vA♭ vB♭ vC vD vE♭	vC minor	vC vD vE♭ vF vG vA♭ vB♭ vC
♭ ♭ (v)	vB♭ major	vB♭ vC vD vE♭ vF vG vA vB♭	vG minor	vG vA vB♭ vC vD vE♭ vF vG
♭ (v)	vF major	vF vG vA vB♭ vC vD vE vF	vD minor	vD vE vF vG vA vB♭ vC vD
(v)	vC major	vC vD vE vF vG vA vB vC	vA minor	vA vB vC vD vE vF vG vA
♯ (v)	vG major	vG vA vB vC vD vE vF♯ vG	vE minor	vE vF♯ vG vA vB vC vD vE
♯ ♯ (v)	vD major	vD vE vF♯ vG vA vB vC♯ vD	vB minor	vB vC♯ vD vE vF♯ vG vA vB
♯ ♯ ♯ (v)	vA major	vA vB vC♯ vD vE vF♯ vG♯ vA	vF♯ minor	vF♯ vG♯ vA vB vC♯ vD vE vF♯
♯ ♯ ♯ ♯ (v)	(vE major)	(vE vF♯ vG♯ vA vB vC♯ vD♯ vE)	(vC♯ minor)	(vC♯ vD♯ vE vF♯ vG♯ vA vB vC♯)

Table 4.21 – 31-edo key signatures using ups and downs, in melodic order

key signature	major key	major scale	minor key	minor scale
no sharps or flats	C major	C D E F G A B C	A minor	A B C D E F G A
(^)	^C major	^C ^D ^E ^F ^G ^A ^B ^C	^A minor	^A ^B ^C ^D ^E ^F ^G ^A
♯ ♯ ♯ ♯ ♯ ♯ ♯ ♭ ♭ ♭ ♭ ♭ (v)	C♯ major vD♭ major	C♯ D♯ E♯ F♯ G♯ A♯ B♯ C♯ vD♭ vE♭ vF vG♭ vA♭ vB♭ vC vD♭	A♯ minor vB♭ minor	A♯ B♯ C♯ D♯ E♯ F♯ G♯ A♯ vB♭ vC vD♭ vE♭ vF vG♭ vA♭ vB♭
♭ ♭ ♭ ♭ ♭	D♭ major	D♭ E♭ F G♭ A♭ B♭ C D♭	B♭ minor	B♭ C D♭ E♭ F G♭ A♭ B♭
♯ ♯ (v)	vD major	vD vE vF♯ vG vA vB vC♯ vD	vB minor	vB vC♯ vD vE vF♯ vG vA vB
♯ ♯	D major	D E F♯ G A B C♯ D	B minor	B C♯ D E F♯ G A B
♯ ♯ (^)	^D major	^D ^E ^F♯ ^G ^A ^B ^C♯ ^D	^B minor	^B ^C♯ ^D ^E ^F♯ ^G ^A ^B
♭ ♭ ♭ (v)	vE♭ major	vE♭ vF vG vA♭ vB♭ vC vD vE♭	vC minor	vC vD vE♭ vF vG vA♭ vB♭ vC
♭ ♭ ♭	E♭ major	E♭ F G A♭ B♭ C D E♭	C minor	C D E♭ F G A♭ B♭ C
♭ ♭ ♭ (^) ♯ ♯ ♯ ♯ (v)	^E♭ major (vE major)	^E♭ ^F ^G ^A♭ ^B♭ ^C ^D ^E♭ (vE vF♯ vG♯ vA vB vC♯ vD♯ vE)	^C minor (vC♯ minor)	^C ^D ^E♭ ^F ^G ^A♭ ^B♭ ^C (vC♯ vD♯ vE vF♯ vG♯ vA vB vC♯)
♯ ♯ ♯ ♯	E major	E F♯ G♯ A B C♯ D♯ E	C♯ minor	C♯ D♯ E F♯ G♯ A B C♯
♯ ♯ ♯ ♯ (^)	^E major	^E ^F♯ ^G♯^A ^B ^C♯ ^D♯ ^E	^C♯ minor	^C♯ ^D♯ ^E ^F♯ ^G♯ ^A ^B ^C♯
♭ (v)	vF major	vF vG vA vB♭ vC vD vE vF	vD minor	vD vE vF vG vA vB♭ vC vD
♭	F major	F G A B♭ C D E F	D minor	D E F G A B♭ C D
♭ (^)	^F major	^F ^G ^A ^B♭ ^C ^D ^E ^F	^D minor	^D ^E ^F ^G ^A ^B♭ ^C ^D
♯ ♯ ♯ ♯ ♯ ♯	F♯ major	F♯ G♯ A♯ B C♯ D♯ E♯ F♯	D♯ minor	D♯ E♯ F♯ G♯ A♯ B C♯ D♯
♭ ♭ ♭ ♭ ♭ ♭	G♭ major	G♭ A♭ B♭ C♭ D♭ E♭ F G♭	E♭ minor	E♭ F G♭ A♭ B♭ C♭ D♭ E♭
♯ (v)	vG major	vG vA vB vC vD vE vF♯ vG	vE minor	vE vF♯ vG vA vB vC vD vE
♯	G major	G A B C D E F♯ G	E minor	E F♯ G A B C D E
♯ (^)	^G major	^G ^A ^B ^C ^D ^E ^F♯ ^G	^E minor	^E ^F♯ ^G ^A ^B ^C ^D ^E
♭ ♭ ♭ ♭ (v)	vA♭ major	vA♭ vB♭ vC vD♭ vE♭ vF vG vA♭	vF minor	vF vG vA♭ vB♭ vC vD♭ vE♭ F
♭ ♭ ♭ ♭	A♭ major	A♭ B♭ C D♭ E♭ F G A♭	F minor	F G A♭ B♭ C D♭ E♭ F
♯ ♯ ♯ (v) ♭ ♭ ♭ ♭ (^)	vA major (^A♭ major)	vA vB vC♯ vD vE vF♯ vG♯ vA (^A♭ ^B♭ ^C ^D♭ ^E♭ ^F ^G ^A♭)	vF♯ minor (^F minor)	vF♯ vG♯ vA vB vC♯ vD vE vF♯ (^F ^G ^A♭ ^B♭ ^C ^D♭ ^E♭ ^F)
♯ ♯ ♯	A major	A B C♯ D E F♯ G♯ A	F♯ minor	F♯ G♯ A B C♯ D E F♯
♯ ♯ ♯ (^)	^A major	^A ^B ^C♯ ^D ^E ^F♯ ^G♯ ^A	^F♯ minor	^F♯ ^G♯ ^A ^B ^C♯ ^D ^E ^F♯
♭ ♭ (v)	vB♭ major	vB♭ vC vD vE♭ vF vG vA vB♭	vG minor	vG vA vB♭ vC vD vE♭ vF vG
♭ ♭	B♭ major	B♭ C D E♭ F G A B♭	G minor	G A B♭ C D E♭ F G
♭ ♭ (^)	^B♭ major	^B♭ ^C ^D ^E♭ ^F ^G ^A ^B♭	^G minor	^G ^A ^B♭ ^C ^D ^E♭ ^F ^G
♯ ♯ ♯ ♯ ♯	B major	B C♯ D♯ E F♯ G♯ A♯ B	G♯ minor	G♯ A♯ B C♯ D♯ E♯ F G♯
♯ ♯ ♯ ♯ ♯ (^) ♭ ♭ ♭ ♭ ♭ ♭ ♭	^B major C♭ major	^B ^C♯ ^D♯ ^E ^F♯ ^G♯ ^A♯ ^B C♭ D♭ E♭ F♭ G♭ A♭ B♭ C♭	^G♯ minor A♭ minor	^G♯ ^A♯ ^B ^C♯ ^D♯ ^E♯ ^F ^G♯ A♭ B♭ C♭ D♭ E♭ F♭ G♭ A♭
(v)	vC major	vC vD vE vF vG vA vB vC	vA minor	vA vB vC vD vE vF vG vA

41-edo: Using only plain tonics isn't recommended. 12 keys require either a global dup/dud or out-of-bounds notes.

Table 4.22 – 41-edo notation (* asterisks indicate the 24 keys that require triple-sharps or triple-flats)

steps	edo cents	interval (with ^v)	major keys	minor keys	interval (no ^v)	major keys	minor keys
0	0¢	P1	**D**	"	P1	**D**	"
1	29	^1	^D	"	-d2	C𝄪	"
2	59	vm2	vE♭	^^D / vE♭	dd3	F♭♭ *	" *
3	88	m2	E♭	vD♯ / E♭	m2	E♭	"
4	117	^m2	^E♭	D♯ / ^E♭	A1	D♯	"
5	146	~2	^^E♭ / vvE	^D♯ / vvE	d34 / -dd2	G♭3 *	C♯3 *
6	176	vM2	vE	"	d3	F♭	"
7	205	M2	**E**	"	M2	**E**	"
8	234	^M2	^E	"	AA1	D𝄪 *	"
9	263	vm3	vF	"	dd4	G♭♭	" *
10	293	m3	**F**	"	m3	**F**	"
11	322	^m3	^F	"	A2	E♯	"
12	351	~3	^^F / vG♭	vvF♯ / ^^F	d35 / A31	A♭3 *	D♯3 *
13	380	vM3	G♭ / vF♯	vF♯	d4	G♭	"
14	410	M3	F♯ / ^G♭	"	M3	F♯	"
15	439	^M3	^F♯ / vvG	"	AA2	E𝄪 *	" *
16	468	v4	vG	"	dd5	A♭♭	"
17	498	P4	**G**	"	P4	**G**	"
18	527	^4	^G	"	A3	F𝄪	"
19	556	~4	vA♭	^^G	dd6	B♭3 *	" *
20	585	vA4 / d5	A♭	vG♯	d5	A♭	"
21	615	A4 / ^d5	^A♭	G♯	A4	G♯	"
22	644	~5	vvA / ^^A♭	^G♯	AA3	F♯3 *	" *
23	673	v5	vA	"	d6	B♭♭	"
24	702	P5	**A**	"	P5	**A**	"
25	732	^5	^A	"	AA4	G𝄪 *	"
26	761	vm6	vB♭	"	dd7	C♭♭	" *
27	790	m6	B♭	"	m6	B♭	"
28	820	^m6	^B♭	^B♭	A5	A♯	"
29	849	~6	^^B♭	vvB	d38 / A34	D♭3 *	G♯3 *
30	878	vM6	vB	"	d7	C♭	"
31	907	M6	**B**	"	M6	**B**	"
32	937	^M6	^B	"	AA5	A𝄪 *	"
33	966	vm7	vC	"	dd8	D♭♭	" *
34	995	m7	**C**	"	m7	**C**	"
35	1024	^m7	^C	"	A6	B♯	"
36	1054	~7	vD♭	^^C / vvC♯	dd9	E♭3 *	" *
37	1083	vM7	D♭	vC♯	d8	D♭	"
38	1112	M7	^D♭	C♯	M7	C♯	"
39	1141	^M7	vvD	^C♯	AA6	B𝄪 *	" *
40	1171	v8	vD	"	d9	E♭♭	"
41	1200	P8	**D**	"	P8	**D**	"

Table 4.23 – 41-edo key signatures using ups and downs, in chain-of-5ths order ("out-of-bounds" notes are underlined)

key signature	major key	major scale	minor key	minor scale
♭ ♭ ♭ ♭ (^^)	(^^A♭ major)	(^^A♭ ^^B♭ ^^C ^^D♭ ^^E♭ ^^F ^^G ^^A♭)	(^^F minor)	(^^F ^^G ^^A♭ ^^B♭ ^^C ^^D♭ ^^E♭ ^^F)
♭ ♭ ♭ (^^)	^^E♭ major	^^E♭ ^^F ^^G ^^A♭ ^^B♭ ^^C ^^D ^^E♭	^^C minor	^^C ^^D ^^E♭ ^^F ^^G ^^A♭ ^^B♭ ^^C
♭ ♭ (^^)	^^B♭ major	^^B♭ ^^C ^^D ^^E♭ ^^F ^^G ^^A ^^B♭	^^G minor	^^G ^^A ^^B♭ ^^C ^^D ^^E♭ ^^F ^^G
♭ (^^)	^^F major	^^F ^^G ^^A ^^B♭ ^^C ^^D ^^E ^^F	^^D minor	^^D ^^E ^^F ^^G ^^A ^^B♭ ^^C ^^D
♭ ♭ ♭ ♭ ♭ ♭ (v)	vG♭ major	vG♭ vA♭ vB♭ vC♭ vD♭ vE♭ vF vG♭	vE♭ minor	vE♭ vF vG♭ vA♭ vB♭ vC♭ vD♭ vE♭
♭ ♭ ♭ ♭ ♭ (v)	vD♭ major	vD♭ vE♭ vF vG♭ vA♭ vB♭ vC vD♭	vB♭ minor	vB♭ vC vD♭ vE♭ vF vG♭ vA♭ vB♭
♭ ♭ ♭ ♭ (v)	vA♭ major	vA♭ vB♭ vC vD♭ vE♭ vF vG vA♭	vF minor	vF vG vA♭ vB♭ vC vD♭ vE♭ vF
♭ ♭ ♭ (v)	vE♭ major	vE♭ vF vG vA♭ vB♭ vC vD vE♭	vC minor	vC vD vE♭ vF vG vA♭ vB♭ vC
♭ ♭ (v)	vB♭ major	vB♭ vC vD vE♭ vF vG vA vB♭	vG minor	vG vA vB♭ vC vD vE♭ vF vG
♭ (v)	vF major	vF vG vA vB♭ vC vD vE vF	vD minor	vD vE vF vG vA vB♭ vC vD
(v)	vC major	vC vD vE vF vG vA vB vC	vA minor	vA vB vC vD vE vF vG vA
♯ (v)	vG major	vG vA vB vC vD vE vF♯ vG	vE minor	vE vF♯ vG vA vB vC vD vE
♯ ♯ (v)	vD major	vD vE vF♯ vG vA vB vC♯ vD	vB minor	vB vC♯ vD vE vF♯ vG vA vB
♯ ♯ ♯ (v)	vA major	vA vB vC♯ vD vE vF♯ vG♯ vA	vF♯ minor	vF♯ vG♯ vA vB vC♯ vD vE vF♯
♯ ♯ ♯ ♯ (v)	vE major	vE vF♯ vG♯vA vB vC♯ vD♯ vE	vC♯ minor	vC♯ vD♯ vE vF♯ vG♯ vA vB vC♯
♯ ♯ ♯ ♯ ♯ (v)	vB major	vB vC♯ vD♯ vE vF♯ vG♯ vA♯ vB	vG♯ minor	vG♯ vA♯ vB vC♯ vD♯ vE♯ vF vG♯
♯ ♯ ♯ ♯ ♯ ♯ (v)	vF♯ major	vF♯ vG♯ vA♯ vB vC♯ vD♯ vE♯ vF♯	vD♯ minor	vD♯ vE♯ vF♯ vG♯ vA♯ vB vC♯ vD♯
♭ ♭ ♭ ♭ ♭ ♭	G♭ major	G♭ A♭ B♭ C♭ D♭ E♭ F G♭	E♭ minor	E♭ F G♭ A♭ B♭ C♭ D♭ E♭
♭ ♭ ♭ ♭ ♭	D♭ major	D♭ E♭ F G♭ A♭ B♭ C D♭	B♭ minor	B♭ C D♭ E♭ F G♭ A♭ B♭
♭ ♭ ♭ ♭	A♭ major	A♭ B♭ C D♭ E♭ F G A♭	F minor	F G A♭ B♭ C D♭ E♭ F
♭ ♭ ♭	E♭ major	E♭ F G A♭ B♭ C D E♭	C minor	C D E♭ F G A♭ B♭ C
♭ ♭	B♭ major	B♭ C D E♭ F G A B♭	G minor	G A B♭ C D E♭ F G
♭	F major	F G A B♭ C D E F	D minor	D E F G A B♭ C D
no sharps / flats	C major	C D E F G A B C	A minor	A B C D E F G A
♯	G major	G A B C D E F♯ G	E minor	E F♯ G A B C D E
♯ ♯	D major	D E F♯ G A B C♯ D	B minor	B C♯ D E F♯ G A B
♯ ♯ ♯	A major	A B C♯ D E F♯ G♯ A	F♯ minor	F♯ G♯ A B C♯ D E F♯
♯ ♯ ♯ ♯	E major	E F♯ G♯ A B C♯ D♯ E	C♯ minor	C♯ D♯ E F♯ G♯ A B C♯
♯ ♯ ♯ ♯ ♯	B major	B C♯ D♯ E F♯ G♯ A♯ B	G♯ minor	G♯ A♯ B C♯ D♯ E♯ F G♯
♯ ♯ ♯ ♯ ♯ ♯	F♯ major	F♯ G♯ A♯ B C♯ D♯ E♯ F♯	D♯ minor	D♯ E♯ F♯ G♯ A♯ B C♯ D♯
♭ ♭ ♭ ♭ ♭ ♭ (^)	^G♭ major	^G♭ ^A♭ ^B♭ ^C♭ ^D♭ ^E♭ ^F ^G♭	^E♭ minor	^E♭ ^F ^G♭ ^A♭ ^B♭ ^C♭ ^D♭ ^E♭
♭ ♭ ♭ ♭ ♭ (^)	^D♭ major	^D♭ ^E♭ ^F ^G♭ ^A♭ ^B♭ ^C ^D♭	^B♭ minor	^B♭ ^C ^D♭ ^E♭ ^F ^G♭ ^A♭ ^B♭
♭ ♭ ♭ ♭ (^)	^A♭ major	^A♭ ^B♭ ^C ^D♭ ^E♭ ^F ^G ^A♭	^F minor	^F ^G ^A♭ ^B♭ ^C ^D♭ ^E♭ F
♭ ♭ ♭ (^)	^E♭ major	^E♭ ^F ^G ^A♭ ^B♭ ^C ^D ^E♭	^C minor	^C ^D ^E♭ ^F ^G ^A♭ ^B♭ ^C
♭ ♭ (^)	^B♭ major	^B♭ ^C ^D ^E♭ ^F ^G ^A ^B♭	^G minor	^G ^A ^B♭ ^C ^D ^E♭ ^F ^G
♭ (^)	^F major	^F ^G ^A ^B♭ ^C ^D ^E ^F	^D minor	^D ^E ^F ^G ^A ^B♭ ^C ^D
(^)	^C major	^C ^D ^E ^F ^G ^A ^B ^C	^A minor	^A ^B ^C ^D ^E ^F ^G ^A
♯ (^)	^G major	^G ^A ^B ^C ^D ^E ^F♯ ^G	^E minor	^E ^F♯ ^G ^A ^B ^C ^D ^E
♯ ♯ (^)	^D major	^D ^E ^F♯ ^G ^A ^B ^C♯ ^D	^B minor	^B ^C♯ ^D ^E ^F♯ ^G ^A ^B
♯ ♯ ♯ (^)	^A major	^A ^B ^C♯ ^D ^E ^F♯ ^G♯ ^A	^F♯ minor	^F♯ ^G♯ ^A ^B ^C♯ ^D ^E ^F♯
♯ ♯ ♯ ♯ (^)	^E major	^E ^F♯ ^G♯ ^A ^B ^C♯ ^D♯ ^E	^C♯ minor	^C♯ ^D♯ ^E ^F♯ ^G♯ ^A ^B ^C♯
♯ ♯ ♯ ♯ ♯ (^)	^B major	^B ^C♯ ^D♯ ^E ^F♯ ^G♯ ^A♯ ^B	^G♯ minor	^G♯ ^A♯ ^B ^C♯ ^D♯ ^E ^F♯ ^G♯
♯ ♯ ♯ ♯ ♯ ♯ (^)	^F♯ major	^F♯ ^G♯ ^A♯ ^B ^C♯ ^D♯ ^E♯ ^F♯	^D♯ minor	^D♯ ^E♯ ^F♯ ^G♯ ^A♯ ^B ^C♯ ^D♯
♯ (vv)	vvG major	vvG vvA vvB vvC vvD vvE vvF♯ vvG	vvE minor	vvE vvF♯ vvG vvA vvB vvC vvD vvE
♯ ♯ (vv)	vvD major	vvD vvE vvF♯ vvG vvA vvB vvC♯ vvD	vvB minor	vvB vvC♯ vvD vvE vvF♯ vvG vvA vvB
♯ ♯ ♯ (vv)	vvA major	vvA vvB vvC♯ vvD vvE vvF♯ vvG♯ vvA	vvF♯ minor	vvF♯ vvG♯ vvA vvB vvC♯ vvD vvE vvF♯
♯ ♯ ♯ ♯ (vv)	(vvE major)	(vvE vvF♯ vvG♯vvA vvB vvC♯ vvD♯ vvE)	(vvC♯ minor)	(vvC♯ vvD♯ vvE vvF♯ vvG♯vvA vvB vvC♯)

Table 4.24 – 41-edo key signatures using ups and downs, in melodic order ("out-of-bounds" notes are underlined)

key signature	major key	major scale	minor key	minor scale
no sharps / flats	C major	C D E F G A B C	A minor	A B C D E F G A
(^)	^C major	^C ^D ^E ^F ^G ^A ^B ^C	^A minor	^A ^B ^C ^D ^E ^F ^G ^A
♭ ♭ ♭ ♭ ♭ (v)	vD♭ major	vD♭ vE♭ vF vG♭ vA♭ vB♭ vC vD♭	vB♭ minor	vB♭ vC vD♭ vE♭ vF vG♭ vA♭ vB♭
♭ ♭ ♭ ♭ ♭	D♭ major	D♭ E♭ F G♭ A♭ B♭ C D♭	B♭ minor	B♭ C D♭ E♭ F G♭ A♭ B♭
♭ ♭ ♭ ♭ ♭ (^)	^D♭ major	^D♭ ^E♭ ^F ^G♭ ^A♭ ^B♭ ^C ^D♭	^B♭ minor	^B♭ ^C ^D♭ ^E♭ ^F ^G♭ ^A♭ ^B♭
♯ ♯ (vv)	vvD major	vvD vvE vvF♯ vvG vvA vvB vvC♯ vvD	vvB minor	vvB vvC♯ vvD vvE vvF♯ vvG vvA vvB
♯ ♯ (v)	vD major	vD vE vF♯ vG vA vB vC♯ vD	vB minor	vB vC♯ vD vE vF♯ vG vA vB
♯ ♯	D major	D E F♯ G A B C♯ D	B minor	B C♯ D E F♯ G A B
♯ ♯ (^)	^D major	^D ^E ^F♯ ^G ^A ^B ^C♯ ^D	^B minor	^B ^C♯ ^D ^E ^F♯ ^G ^A ^B
♭ ♭ ♭ (v)	vE♭ major	vE♭ vF vG vA♭ vB♭ vC vD vE♭	vC minor	vC vD vE♭ vF vG vA♭ vB♭ vC
♭ ♭ ♭	E♭ major	E♭ F G A♭ B♭ C D E♭	C minor	C D E♭ F G A♭ B♭ C
♭ ♭ ♭ (^)	^E♭ major	^E♭ ^F ^G ^A♭ ^B♭ ^C ^D ^E♭	^C minor	^C ^D ^E♭ ^F ^G ^A♭ ^B♭ ^C
♭ ♭ ♭ (^^) ♯ ♯ ♯ ♯ (vv)	^^E♭ major (vvE major)	^^E♭ ^^F ^^G ^^A♭ ^^B♭ ^^C ^^D ^^E♭ (vvE vvF♯ vvG♯vvA vvB vvC♯ vvD♯ vvE)	^^C minor (vvC♯ minor)	^^C ^^D ^^E♭ ^^F ^^G ^^A♭ ^^B♭ ^^C (vvC♯ vvD♯ vvE vvF♯ vvG♯vvA vvB vvC♯)
♯ ♯ ♯ ♯ (v)	vE major	vE vF♯ vG♯vA vB vC♯ vD♯ vE	vC♯ minor	vC♯ vD♯ vE vF♯ vG♯ vA vB vC♯
♯ ♯ ♯ ♯	E major	E F♯ G♯ A B C♯ D♯ E	C♯ minor	C♯ D♯ E F♯ G♯ A B C♯
♯ ♯ ♯ ♯ (^)	^E major	^E ^F♯ ^G♯ ^A ^B ^C♯ ^D♯ ^E	^C♯ minor	^C♯ ^D♯ ^E ^F♯ ^G♯ ^A ^B ^C♯
♭ (v)	vF major	vF vG vA vB♭ vC vD vE vF	vD minor	vD vE vF vG vA vB♭ vC vD
♭	F major	F G A B♭ C D E F	D minor	D E F G A B♭ C D
♭ (^)	^F major	^F ^G ^A ^B♭ ^C ^D ^E ^F	^D minor	^D ^E ^F ^G ^A ^B♭ ^C ^D
♭ (^^)	^^F major	^^F ^^G ^^A ^^B♭ ^^C ^^D ^^E ^^F	^^D minor	^^D ^^E ^^F ^^G ^^A ^^B♭ ^^C ^^D
♭ ♭ ♭ ♭ ♭ ♭ (v)	vG♭ major	vG♭ vA♭ vB♭ **vC♭** vD♭ vE♭ vF vG♭	vE♭ minor	vE♭ vF vG♭ vA♭ vB♭ **vC♭** vD♭ vE♭
♯ ♯ ♯ ♯ ♯ ♯ (v)	vF♯ major	vF♯ vG♯ vA♯ vB vC♯ vD♯ **vE♯** vF♯	vD♯ minor	vD♯ **vE♯** vF♯ vG♯ vA♯ vB vC♯ vD♯
♭ ♭ ♭ ♭ ♭ ♭	G♭ major	G♭ A♭ B♭ **C♭** D♭ E♭ F G♭	E♭ minor	E♭ F G♭ A♭ B♭ **C♭** D♭ E♭
♯ ♯ ♯ ♯ ♯ ♯	F♯ major	F♯ G♯ A♯ B C♯ D♯ **E♯** F♯	D♯ minor	D♯ **E♯** F♯ G♯ A♯ B C♯ D♯
♭ ♭ ♭ ♭ ♭ ♭ (^)	^G♭ major	^G♭ ^A♭ ^B♭ **^C♭** ^D♭ ^E♭ ^F ^G♭	^E♭ minor	^E♭ ^F ^G♭ ^A♭ ^B♭ **^C♭** ^D♭ ^E♭
♯ ♯ ♯ ♯ ♯ ♯ (^) ♯ (vv)	^F♯ major vvG major	^F♯ ^G♯ ^A♯ ^B ^C♯ ^D♯ **^E♯** ^F♯ vvG vvA vvB vvC vvD vvE vvF♯ vvG	^D♯ minor vvE minor	^D♯ **^E♯** ^F♯ ^G♯ ^A♯ ^B ^C♯ ^D♯ vvE vvF♯ vvG vvA vvB vvC vvD vvE
♯ (v)	vG major	vG vA vB vC vD vE vF♯ vG	vE minor	vE vF♯ vG vA vB vC vD vE
♯	G major	G A B C D E F♯ G	E minor	E F♯ G A B C D E
♯ (^)	^G major	^G ^A ^B ^C ^D ^E ^F♯ ^G	^E minor	^E ^F♯ ^G ^A ^B ^C ^D ^E
♭ ♭ ♭ ♭ (v)	vA♭ major	vA♭ vB♭ vC vD♭ vE♭ vF vG vA♭	vF minor	vF vG vA♭ vB♭ vC vD♭ vE♭ vF
♭ ♭ ♭ ♭	A♭ major	A♭ B♭ C D♭ E♭ F G A♭	F minor	F G A♭ B♭ C D♭ E♭ F
♭ ♭ ♭ ♭ (^)	^A♭ major	^A♭ ^B♭ ^C ^D♭ ^E♭ ^F ^G ^A♭	^F minor	^F ^G ^A♭ ^B♭ ^C ^D♭ ^E♭ F
♯ ♯ ♯ (vv) ♭ ♭ ♭ ♭ (^^)	vvA major (^^A♭ major)	vvA vvB vvC♯ vvD vvE vvF♯ vvG♯ vvA (^^A♭ ^^B♭ ^^C ^^D♭ ^^E♭ ^^F ^^G ^^A♭)	vvF♯ minor (^^F minor)	vvF♯ vvG♯ vvA vvB vvC♯ vvD vvE vvF♯ (^^F ^^G ^^A♭ ^^B♭ ^^C ^^D♭ ^^E♭ ^^F)
♯ ♯ ♯ (v)	vA major	vA vB vC♯ vD vE vF♯ vG♯ vA	vF♯ minor	vF♯ vG♯ vA vB vC♯ vD vE vF♯
♯ ♯ ♯	A major	A B C♯ D E F♯ G♯ A	F♯ minor	F♯ G♯ A B C♯ D E F♯
♯ ♯ ♯ (^)	^A major	^A ^B ^C♯ ^D ^E ^F♯ ^G♯ ^A	^F♯ minor	^F♯ ^G♯ ^A ^B ^C♯ ^D ^E ^F♯
♭ ♭ (v)	vB♭ major	vB♭ vC vD vE♭ vF vG vA vB♭	vG minor	vG vA vB♭ vC vD vE♭ vF vG
♭ ♭	B♭ major	B♭ C D E♭ F G A B♭	G minor	G A B♭ C D E♭ F G
♭ ♭ (^)	^B♭ major	^B♭ ^C ^D ^E♭ ^F ^G ^A ^B♭	^G minor	^G ^A ^B♭ ^C ^D ^E♭ ^F ^G
♭ ♭ (^^)	^^B♭ major	^^B♭ ^^C ^^D ^^E♭ ^^F ^^G ^^A ^^B♭	^^G minor	^^G ^^A ^^B♭ ^^C ^^D ^^E♭ ^^F ^^G
♯ ♯ ♯ ♯ ♯ (v)	vB major	vB vC♯ vD♯ vE vF♯ vG♯ vA♯ vB	vG♯ minor	vG♯ vA♯ vB vC♯ vD♯ vE♯ vF vG♯
♯ ♯ ♯ ♯ ♯	B major	B C♯ D♯ E F♯ G♯ A♯ B	G♯ minor	G♯ A♯ B C♯ D♯ E♯ F G♯
♯ ♯ ♯ ♯ ♯ (^)	^B major	^B ^C♯ ^D♯ ^E ^F♯ ^G♯ ^A♯ ^B	^G♯ minor	^G♯ ^A♯ ^B ^C♯ ^D♯ ^E ^F♯ ^G♯
(v)	vC major	vC vD vE vF vG vA vB vC	vA minor	vA vB vC vD vE vF vG vA

Pentatonic edos: In edos 5, 10, 15, 20, 25 & 30, E and F are the same note, just like C♯ and D♭ are in 12-edo. It's E in some contexts, but F in others. B and C are also the same. The minor 2nd is a unison, which means that the major 2nd is also a minor 3rd, the major 3rd is also a 4th, etc. In 15-edo, every key has at least three names:

Table 4.25 – Note names and interval names in 15-edo

C♯ D E♭	^C♯ ^D ^E♭	vD♯ vE vF vG♭	D♯ E F G♭	^D♯ ^E ^F ^G♭	vF♯ vG vA♭	F♯ G A♭	^F♯ ^G ^A♭	vG♯ vA vB♭	G♯ A B♭	^G♯ ^A ^B♭	vA♯ vB vC vD♭	A♯ B C D♭	^A♯ ^B ^C ^D♭	vC♯ vD vE♭	C♯ D E♭
P1 m2	^1 ^m2	vA1 vM2 vm3	A1 M2 m3	^M2 ^m3	vM3 v4	M3 P4 d5	^M3 ^4 ^d5	vA4 v5 vm6	A4 P5 m6	^5 ^m6	vM6 vm7	M6 m7 d8	^M6 ^m7 ^d8	vM7 v8	M7 P8

Normally, a key signature implies a default heptatonic scale. But in a pentatonic edo, every conventional key signature implies the exact same five notes. Nevertheless, key signatures can indicate the tonic and minimize accidentals. For example, the D downmajor scale can be notated as an altered D major: D E vF♯ G A vB vC♯ D. The key signature has the usual two sharps, and using it avoids having to sharpen the F and C repeatedly. However, F, B and C do need to be downed repeatedly. Analogous to a color signature, one might use an ups/downs signature reading vF♯ vB vC♯. This would be placed immediately above the key signature on every line.

^D downmajor as an altered ^D major is ^D ^E F♯ ^G ^A B C♯ ^D. The key signature has two sharps and a global up. Both ^F♯ and ^C♯ need to be made plain (not upped) repeatedly with a sharp sign, which cancels the implied up. Likewise, B would need a natural sign. vD downmajor as an altered vD major is vD vE vvF♯ vG vA vvB vvC♯ vD. The key signature is two sharps and a global down. Duds could be avoided by notating it as an altered vD minor: vD vE ^F vG vA ^B ♭ ^C vD. However, downmajor intervals like vD – vvF♯ would be written as dupminor intervals vD – ^F, which could be misleading. Another possibility is ^^C major.

D upminor is notated as an altered D minor: D E ^F G A ^B♭ ^C D. The F, B and C need to be upped repeatedly. ^D upminor would be an altered ^D minor, with a key signature of one flat and a global up: ^D ^E ^^F ^G ^A ^^B♭ ^^C ^D.

Table 4.26 – Preferred tonic names for 15-edo

downmajor	D	^D	vE/vF	E/F	^E/^F	vG	G	^G	vA	A	^A	vC	C	^C	vD
upminor	"	"	vE	E	^E	"	"	"	"	"	"	vB/vC	B/C	^B/^C	"

In 15-edo, the D Triyo (porcupine) scale can be notated as an altered D minor: D vE ^F G A ^^B♭ ^C D. Some downmajor 2nds are inevitably spelled as dupminor 2nds. Notating it as an altered D dorian scale would avoid dups (vB not ^^B♭), but the key signature would no longer indicate that D is the tonic. ^D Triyo as an altered ^D major is ^D E vF♯ ^G ^A B vC♯ ^D. E and B need natural signs to cancel the ups, and F♯ and C♯ need downs.

D pentatonic could be notated as D major with 2 enhamonic equivalents: D E F♯=G A B C♯=D. The 5-edo+yo (blackwood) scale could be notated as an altered major scale with 3 added notes. D 5-edo+yo[10] would be D vE E vF♯ G vA A vB B vC♯ D. The key signature for both scales would be the usual 2 sharps.

Table 4.27 – The 15-edo key signatures, in melodic order

major key	major scale	key signature	minor key	minor scale
D major	D E F♯ G A B C♯ D	♯ ♯ ♭ ♭ ♭	B minor (C minor)	B C♯ D E F♯ G A B (C D E♭ F G A♭ B♭ C)
^D major	^D ^E ^F♯ ^G ^A ^B ^C♯ ^D	♯ ♯ (^) ♭ ♭ ♭ (^)	^B minor (^C minor)	^B ^C♯ ^D ^E ^F♯ ^G ^A ^B (^C ^D ^E♭ ^F ^G ^A♭ ^B♭ ^C)
(vE major) vF major	(vE vF♯ vG♯ vA vB vC♯ vD♯ vE) vF vG vA vB♭ vC vD vE vF	♯ ♯ ♯ ♯ (v) ♭ (v)	vD minor	vD vE vF vG vA vB♭ vC vD
(E major) F major	(E F♯ G♯ A B C♯ D♯ E) F G A B♭ C D E F	♯ ♯ ♯ ♯ ♭	D minor	D E F G A B♭ C D
(^E major) ^F major	(^E ^F♯ ^G♯ ^A ^B ^C♯ ^D♯ ^E) ^F ^G ^A ^B♭ ^C ^D ^E ^F	♯ ♯ ♯ ♯ (^) ♭ (^)	^D minor	^D ^E ^F ^G ^A ^B♭ ^C ^D
vG major	vG vA vB vC vD vE vF♯ vG	♯ (v) ♭ ♭ ♭ ♭ (v)	vE minor (vF minor)	vE vF♯ vG vA vB vC vD vE (vF vG vA♭ vB♭ vC vD♭ vE♭ vF)
G major	G A B C D E F♯ G	♯ ♭ ♭ ♭ ♭	E minor (F minor)	E F♯ G A B C D E (F G A♭ B♭ C D♭ E♭ F)
^G major	^G ^A ^B ^C ^D ^E ^F♯ ^G	♯ (^) ♭ ♭ ♭ ♭ (v)	^E minor (^F minor)	^E ^F♯ ^G ^A ^B ^C ^D ^E (^F ^G ^A♭ ^B♭ ^C ^D♭ ^E♭ ^F)
vA major	vA vB vC♯ vD vE vF♯ vG♯ vA	♯ ♯ ♯ (v)	vG minor	vG vA vB♭ vC vD vE♭ vF vG
A major	A B C♯ D E F♯ G♯ A	♯ ♯ ♯	G minor	G A B♭ C D E♭ F G
^A major	^A ^B ^C♯ ^D ^E ^F♯ ^G♯ ^A	♯ ♯ ♯ (^)	^G minor	^G ^A ^B♭ ^C ^D ^E♭ ^F ^G
(vB major) vC major	(vB vC♯ vD♯ vE vF♯ vG♯ vA♯ vB) vC vD vE vF vG vA vB vC	♯ ♯ ♯ ♯ ♯ (v) (v)	vA minor	vA vB vC vD vE vF vG vA
(B major) C major	(B C♯ D♯ E F♯ G♯ A♯ B) C D E F G A B C	♯ ♯ ♯ ♯ ♯ no sharps or flats	A minor	A B C D E F G A
(^B major) ^C major	(^B ^C♯ ^D♯ ^E ^F♯ ^G♯ ^A♯ ^B) ^C ^D ^E ^F ^G ^A ^B ^C	♯ ♯ ♯ ♯ ♯ (^) (^)	^A minor	^A ^B ^C ^D ^E ^F ^G ^A
vD major	vD vE vF♯ vG vA vB vC♯ vD	♯ ♯ (v) ♭ ♭ ♭ ♭ (v)	vB minor (vC minor)	vB vC♯ vD vE vF♯ vG vA vB (vC vD vE♭ vF vG vA♭ vB♭ vC)

The key signature could specify a downmajor or upminor scale, thus implying a heptatonic scale. Ups and downs could be placed as sharps and flats would be, either by themselves or immediately preceding a sharp/flat. Natural signs could be used as placeholders, to preserve the familiar shape of key signatures. But the key signatures would still be difficult to sight-read, as seen below. Since upminor isn't a mode of downmajor, the upminor key signatures would all be different. Thus there would be 30 possible key signatures. Better to use an ups/downs signature above the key signature.

Table 4.28 – The 15-edo downmajor key signatures, in melodic order (not recommended)

key signature	downmajor key	downmajor scale
v♯ v♯ ♮ ♮ ♮ ♮ v	D downmajor	D E vF♯ G A vB vC♯ D
♯ ♯ ^ ^ ^ ^	^D downmajor	^D ^E F♯ ^G ^A B C♯ ^D
♯ v♯ v♯ v♯ (v) ♭ v v v (v)	vE downmajor vF downmajor	vE vF♯ vvG♯ vA vB vvC♯ vvD♯ vE vF vG vvA vB♭ vC vvD vvE vF
♯ v♯ v♯ v♯ ♭ v v v	E downmajor F downmajor	E F♯ vG♯ A B vC♯ vD♯ E F G vA B♭ C vD vE F
^♯ ♯ ♯ ♯ ^ ^ ^ ^♭ ♮ ♮ ♮ ^ ^ ^	^E downmajor ^F downmajor	^E ^F♯ G♯ ^A ^B C♯ D♯ ^E ^F ^G A ^B♭ ^C D E ^F
vv♯ v v v v vv vv	vG downmajor	vG vA vvB vC vD vvE vvF♯ vG
v♯ ♮ ♮ ♮ ♮ v v	G downmajor	G A vB C D vE vF♯ G
♯ ^ ^ ^ ^	^G downmajor	^G ^A B ^C ^D E F♯ ^G
v♯ v♯ v♯ (v)	vA downmajor	vA vB vvC♯ vD vE vvF♯ vvG♯ vA
v♯ v♯ v♯	A downmajor	A B vC♯ D E vF♯ vG♯ A
♯ ♯ ♯ ^ ^ ^ ^	^A downmajor	^A ^B C♯ ^D ^E F♯ G♯ ^A
♯ ♯ v♯ v♯ v♯ (v) v v v (v) (placed as flats)	vB downmajor vC downmajor	vB vC♯ vvD♯ vE vF♯ vvG♯ vvA♯ vB vC vD vvE vF vG vvA vvB vC
♯ ♯ v♯ v♯ v♯ v v v (placed as flats)	B downmajor C downmajor	B C♯ vD♯ E F♯ vG♯ vA♯ B C D vE F G vA vB C
^♯ ^♯ ♯ ♯ ♯ ^ ^ ^ ^ ^ ^ (placed as sharps)	^B downmajor ^C downmajor	^B ^C♯ D♯ ^E ^F♯ G♯ A♯ ^B ^C ^D E ^F ^G A B ^C
♯ ♯ v v v v	vD downmajor	vD vE vvF♯ vG vA vvB vvC♯ vD

Perfect edos (7, 14, 21, 28 & 35) don't require sharps or flats. There is no difference between major and minor keys, and all keys are perfect. Both C perfect and D perfect use the same 7 notes. However, if desired, sharps and flats may be used in the key signature only to indicate the tonic. 14-edo has 7 plain keys and 7 up keys (which can also be written as down keys). 21-edo has 7 plain, 7 up and 7 down keys. The key signature for non-plain keys contains a global-up or a global-down. In an up key, a natural sign serves to cancel the implied up.

28-edo and 35-edo have dup and dud keys. In a dup key, the key signature contains a global dup. In such a key, a single up sign indicates an up note, and a natural sign indicates a plain (not upped) note.

Table 4.29 – 21-edo key signatures, in melodic order

key signature	perfect key	perfect scale
no sharps or flats	C perfect	C D E F G A B C
(^)	^C perfect	^C ^D ^E ^F ^G ^A ^B ^C
♯ ♯ (v)	vD perfect	vD vE vF vG vA vB vC vD
♯ ♯	D perfect	D E F G A B C D
♯ ♯ (^)	^D perfect	^D ^E ^F ^G ^A ^B ^C ^D
♯ ♯ ♯ ♯ (v)	vE perfect	vE vF vG vA vB vC vD vE
♯ ♯ ♯ ♯	E perfect	E F G A B C D E
♯ ♯ ♯ ♯ (^)	^E perfect	^E ^F ^G ^A ^B ^C ^D ^E
♭ (v)	vF perfect	vF vG vA vB vC vD vE vF
♭	F perfect	F G A B C D E F
♭ (^)	^F perfect	^F ^G ^A ^B ^C ^D ^E ^F
♯ (v)	vG perfect	vG vA vB vC vD vE vF vG
♯	G perfect	G A B C D E F G
♯ (^)	^G perfect	^G ^A ^B ^C ^D ^E ^F ^G
♯ ♯ ♯ (v)	vA perfect	vA vB vC vD vE vF vG vA
♯ ♯ ♯	A perfect	A B C D E F G A
♯ ♯ ♯ (^)	^A perfect	^A ^B ^C ^D ^E ^F ^G ^A
♯ ♯ ♯ ♯ ♯ (v)	vB perfect	vB vC vD vE vF vG vA vB
♯ ♯ ♯ ♯ ♯	B perfect	B C D E F G A B
♯ ♯ ♯ ♯ ♯ (^)	^B perfect	^B ^C ^D ^E ^F ^G ^A ^B
(v)	vC perfect	vC vD vE vF vG vA vB vC

Superflat edos: In edos 9, 11, 13b, 16, 18b and 23, major and minor are reversed. The major scale runs ssLsssL and sounds gu, and the minor scale runs sLssLss and sounds yo (see appendix for the definition of yo and gu).

Table 4.30 – Preferred tonic names for 9-edo

major	C	D	E	E♭	F	G	A	B/A♭	B♭
minor	"	"	"	F♯	"	"	"	B	C♯

Table 4.31 – Preferred tonic names for 11-edo

with ^v	major	C	D	E	^E	vF	F	G	A	B	^B	vC
	minor	"	"	"	"	"	"	"	"	"	"	"
without ^v	major	C	D	E	D♭	E♭	F	G	A	B	A♭	B♭
	minor	"	"	"	F♯	G♯	"	"	"	"	C♯	"

Table 4.32 – Preferred tonic names for 13b-edo

with ^v	major	C	D	E	^E	^^E/vvF	vF	F	G	A	B	^B	^^B/vvC	vC
	minor	"	"	"	"	"	"	"	"	"	"	"	"	"
without ^v	major	C	D	E	F♯	D♭	E♭	F	G	A	B	G♭	A♭	B♭
	minor	"	"	"	F♯	G♯	"	"	"	"	"	C♯	D♯	"

Table 4.33 – Preferred tonic names for 16-edo

major	C	C♭	D	D♭	E	E♭	F♯	F	F♭/G♯	G	G♭	A	A♭	B	B♭	C♯
minor	"	D♯	"	D♭/E♯	"	"	"	"	G♯	"	A♯	"	"	"	"	"

Table 4.34 – The 16-edo key signatures, in chain-of-fifths order

key signature	major key	major scale	minor key	minor scale
(𝄫 𝄫 𝄫 ♭ ♭ ♭ ♭)			(C♭ minor)	(C♭ D♭ E♭♭ F♭ G♭ A♭♭ B♭♭ C♭)
(𝄫 𝄫 ♭ ♭ ♭ ♭ ♭)			(G♭ minor)	(G♭ A♭ B♭♭ C♭ D♭ E♭♭ F♭ G♭)
𝄫 ♭ ♭ ♭ ♭ ♭ ♭	F♭ major	F♭ G♭ A♭ B♭♭ C♭ D♭ E♭ F♭	D♭ minor	D♭ E♭ F♭ G♭ A♭ B♭♭ C♭ D♭
♭ ♭ ♭ ♭ ♭ ♭ ♭	C♭ major	C♭ D♭ E♭ F♭ G♭ A♭ B♭ C♭	A♭ minor	A♭ B♭ C♭ D♭ E♭ F♭ G♭ A♭
♭ ♭ ♭ ♭ ♭ ♭	G♭ major	G♭ A♭ B♭ C♭ D♭ E♭ F G♭	E♭ minor	E♭ F G♭ A♭ B♭ C♭ D♭ E♭
♭ ♭ ♭ ♭ ♭	D♭ major	D♭ E♭ F G♭ A♭ B♭ C D♭	B♭ minor	B♭ C D♭ E♭ F G♭ A♭ B♭
♭ ♭ ♭ ♭	A♭ major	A♭ B♭ C D♭ E♭ F G A♭	F minor	F G A♭ B♭ C D♭ E♭ F
♭ ♭ ♭	E♭ major	E♭ F G A♭ B♭ C D E♭	C minor	C D E♭ F G A♭ B♭ C
♭ ♭	B♭ major	B♭ C D E♭ F G A B♭	G minor	G A B♭ C D E♭ F G
♭	F major	F G A B♭ C D E F	D minor	D E F G A B♭ C D
no sharps or flats	C major	C D E F G A B C	A minor	A B C D E F G A
♯	G major	G A B C D E F♯ G	E minor	E F♯ G A B C D E
♯ ♯	D major	D E F♯ G A B C♯ D	B minor	B C♯ D E F♯ G A B
♯ ♯ ♯	A major	A B C♯ D E F♯ G♯ A	F♯ minor	F♯ G♯ A B C♯ D E F♯
♯ ♯ ♯ ♯	E major	E F♯ G♯ A B C♯ D♯ E	C♯ minor	C♯ D♯ E F♯ G♯ A B C♯
♯ ♯ ♯ ♯ ♯	B major	B C♯ D♯ E F♯ G♯ A♯ B	G♯ minor	G♯ A♯ B C♯ D♯ E♯ F G♯
♯ ♯ ♯ ♯ ♯ ♯	F♯ major	F♯ G♯ A♯ B C♯ D♯ E♯ F♯	D♯ minor	D♯ E♯ F♯ G♯ A♯ B C♯ D♯
♯ ♯ ♯ ♯ ♯ ♯ ♯	C♯ major	C♯ D♯ E♯ F♯ G♯ A♯ B♯ C♯	A♯ minor	A♯ B♯ C♯ D♯ E♯ F♯ G♯ A♯
X ♯ ♯ ♯ ♯ ♯ ♯	G♯ major	G♯ A♯ B♯ C♯ D♯ E♯ F𝄪 G♯	E♯ minor	E♯ F𝄪 G♯ A♯ B♯ C♯ D♯ E♯
(X X ♯ ♯ ♯ ♯ ♯)	(D♯ major)	(D♯ E♯ F𝄪 G♯ A♯ B♯ C𝄪 D♯)		
(X X X ♯ ♯ ♯ ♯)	(A♯ major)	(A♯ B♯ C𝄪 D♯ E♯ F𝄪 G𝄪 A♯)		

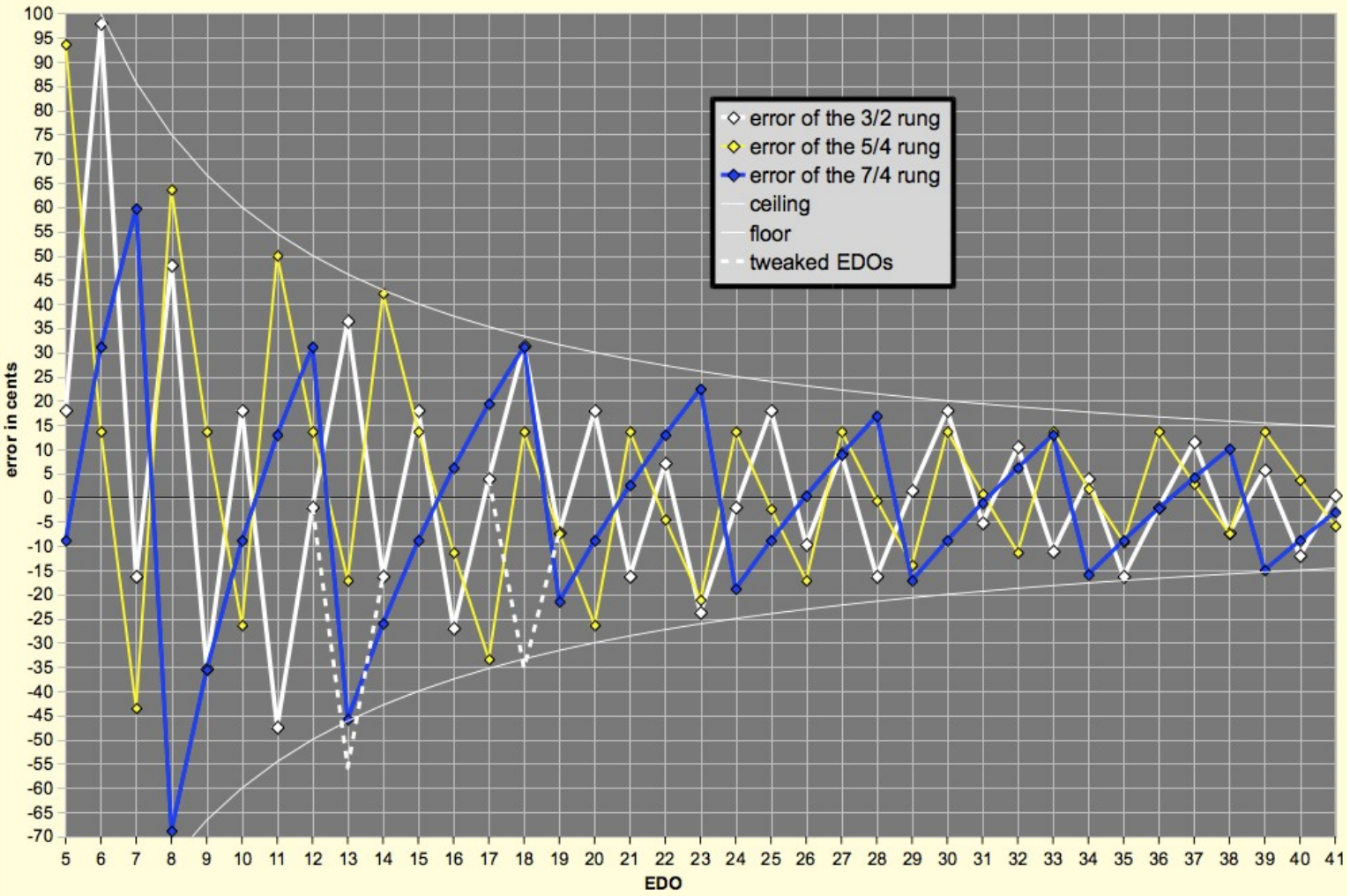
Table 4.35 – The 16-edo key signatures, in melodic order

major key	major scale	key signature	minor key	minor scale
C major	C D E F G A B C	no sharps or flats	A minor	A B C D E F G A
C♭ major (D♯ major)	C♭ D♭ E♭ F♭ G♭ A♭ B♭ C♭ (D♯ E♯ F𝄪 G♯ A♯ B♯ C𝄪 D♯)	♭ ♭ ♭ ♭ ♭ ♭ ♭ (𝄪 𝄪 ♯ ♯ ♯ ♯ ♯)	A♭ minor	A♭ B♭ C♭ D♭ E♭ F♭ G♭ A♭
D major	D E F♯ G A B C♯ D	♯ ♯	B minor	B C♯ D E F♯ G A B
D♭ major	D♭ E♭ F G♭ A♭ B♭ C D♭	♭ ♭ ♭ ♭ ♭	B♭ minor	B♭ C D♭ E♭ F G♭ A♭ B♭
E major	E F♯ G♯ A B C♯ D♯ E	♯ ♯ ♯ ♯	C♯ minor	C♯ D♯ E F♯ G♯ A B C♯
E♭ major	E♭ F G A♭ B♭ C D E♭	♭ ♭ ♭	C minor	C D E♭ F G A♭ B♭ C
F♯ major	F♯ G♯ A♯ B C♯ D♯ E♯ F♯	♯ ♯ ♯ ♯ ♯ ♯ (𝄫 𝄫 𝄫 ♭ ♭ ♭ ♭)	D♯ minor (C♭ minor)	D♯ E♯ F♯ G♯ A♯ B C♯ D♯ (C♭ D♭ E♭♭ F♭ G♭ A♭♭ B♭♭ C♭)
F major	F G A B♭ C D E F	♭	D minor	D E F G A B♭ C D
F♭ major G♯ major	F♭ G♭ A♭ B♭♭ C♭ D♭ E♭ F♭ G♯ A♯ B♯ C♯ D♯ E♯ F𝄪 G♯	𝄫 ♭ ♭ ♭ ♭ ♭ ♭ 𝄪 ♯ ♯ ♯ ♯ ♯ ♯	D♭ minor E♯ minor	D♭ E♭ F♭ G♭ A♭ B♭♭ C♭ D♭ E♯ F𝄪 G♯ A♯ B♯ C♯ D♯ E♯
G major	G A B C D E F♯ G	♯	E minor	E F♯ G A B C D E
G♭ major (A♯ major)	G♭ A♭ B♭ C♭ D♭ E♭ F G♭ (A♯ B♯ C𝄪 D♯ E♯ F𝄪 G𝄪 A♯)	♭ ♭ ♭ ♭ ♭ ♭ (𝄪 𝄪 𝄪 ♯ ♯ ♯ ♯)	E♭ minor	E♭ F G♭ A♭ B♭ C♭ D♭ E♭
A major	A B C♯ D E F♯ G♯ A	♯ ♯ ♯	F♯ minor	F♯ G♯ A B C♯ D E F♯
A♭ major	A♭ B♭ C D♭ E♭ F G A♭	♭ ♭ ♭ ♭	F minor	F G A♭ B♭ C D♭ E♭ F
B major	B C♯ D♯ E F♯ G♯ A♯ B	♯ ♯ ♯ ♯ ♯	G♯ minor	G♯ A♯ B C♯ D♯ E♯ F G♯
B♭ major	B♭ C D E♭ F G A B♭	♭ ♭	G minor	G A B♭ C D E♭ F G
C♯ major	C♯ D♯ E♯ F♯ G♯ A♯ B♯ C♯	♯ ♯ ♯ ♯ ♯ ♯ ♯ (𝄫 𝄫 ♭ ♭ ♭ ♭ ♭)	A♯ minor (G♭ minor)	A♯ B♯ C♯ D♯ E♯ F♯ G♯ A♯ (G♭ A♭ B♭♭ C♭ D♭ E♭♭ F♭ G♭)

Section 5 – Plotting JI Approximations On the Scale Tree

This section discusses how well each edo represents just intonation for various prime limits. It's about how to choose an edo, rather than how to notate it. The next chart shows how accurately each edo from 5-edo to 41-edo approximates 3/2, 5/4 and 7/4. The curved lines represent the maximum possible discrepancy of half an edostep. It includes 13b-edo and 18b-edo along with 13-edo and 18-edo.

Figure 5.1 – The discrepancy (sharp or flat) of 3/2, 5/4 and 7/4 in each edo from 5 to 41



In the next figure, green and red lines have been added to the scale tree, showing how well each edo approximates 5/4. Green lines represent minimum discrepancy. The edo's discrepancy equals four times the edo's distance from the nearest green line. For example, 12-edo is 3.5¢ to the right of a green line, and 12-edo's approximation of 5/4 is 14¢ sharp. This distance is always measured horizontally. 8-edo is a full 16¢ away from the green line, even though it's not far below it.

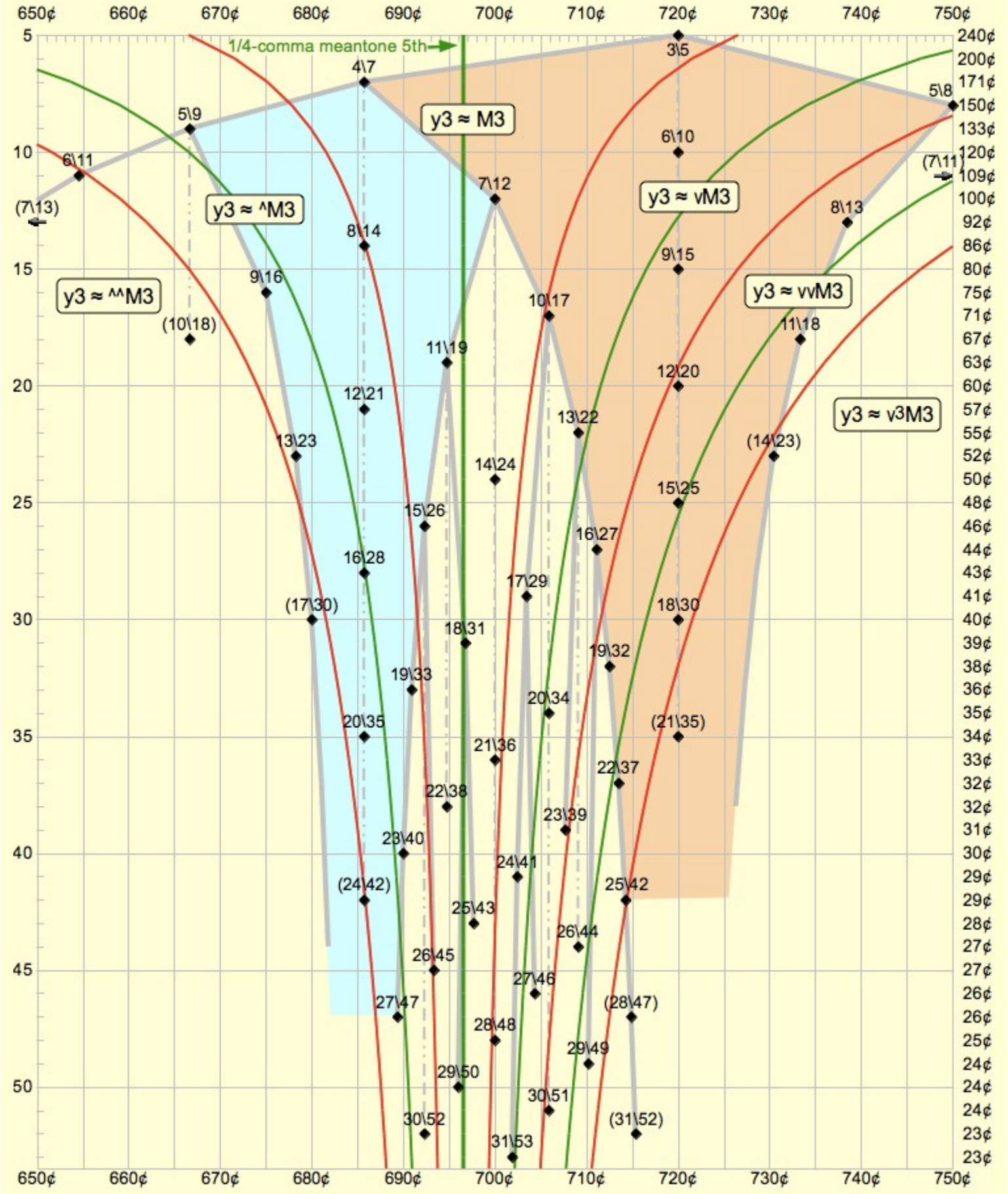
Red lines represent maximum discrepancy. If an edo falls near a red line, like 14-edo and 17-edo, 5/4 falls almost exactly between two edo notes.

The red lines define regions of the scale tree that correspond to 81/80's keyspan (size in edosteps). The edos that support meantone temperament are all contained in the region of the scale tree marked "y3 ≈ M3". (y3 = yo or yellow 3rd = 5/4, see appendix.) The major 3rd is defined as four 5ths minus two octaves, even for superflat edos.

In edos like 15 or 22, 81/80 maps to 1 edostep, and 5/4 ≈ vM3, a downmajor 3rd. In edos like 16 or 21, 81/80 is a descending edostep, and 5/4 ≈ ^M3. Upmajor 3rd is used loosely here to mean the interval one edostep wider than the major 3rd. In many edos in the 5/4 ≈ ^M3 category, this interval actually has a different name: ^3, A3, or even m3.

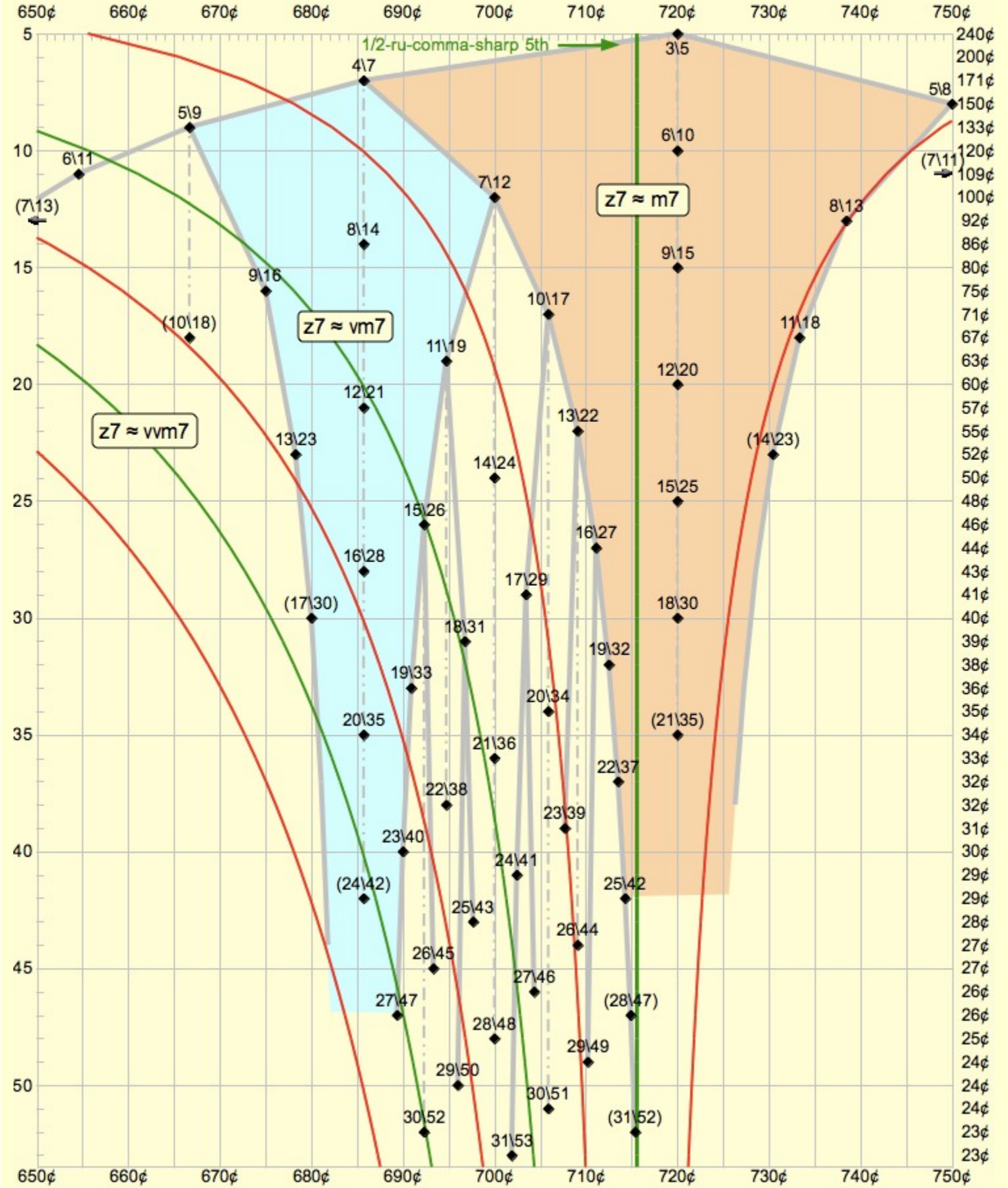
The straight green line is the quarter-comma meantone fifth = 696.6¢. The red and green boundary lines are 1/8 of an edostep apart. For example, 15-edo has an 80¢ edostep, and the boundary lines cross the horizontal 15-edo line at points 10¢ apart. Each edo's step size is listed on the right.

Figure 5.2 – The discrepancy of the yo 3rd 5/4 and the keyspan of the gu comma 81/80 plotted onto the scale tree



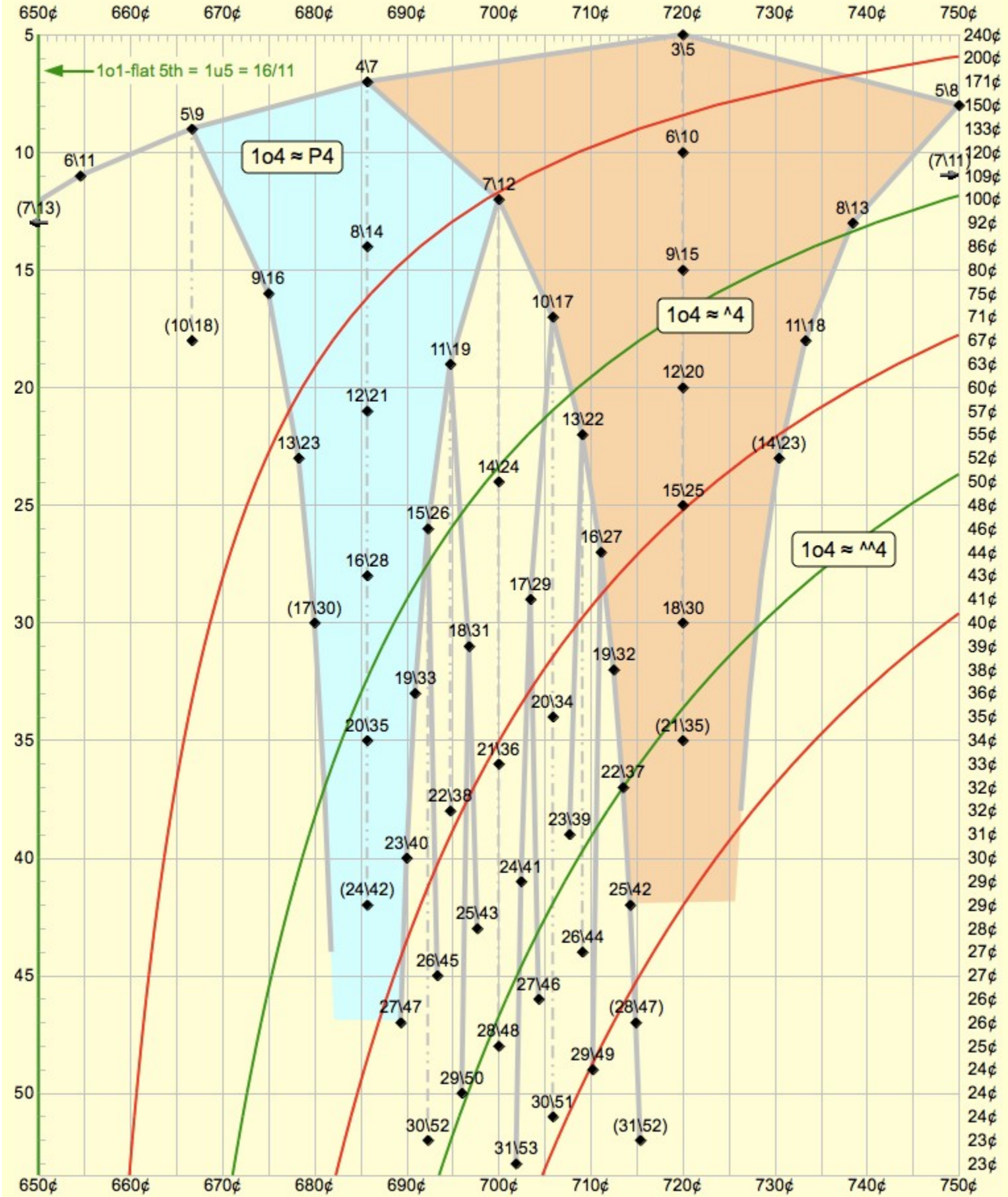
An edo's discrepancy for 7/4 (zo 7th) is twice the distance from the nearest green line. The straight green line is a 5th that's half of 64/63 sharp. Because it's sharp, if the edo falls to the right of a green line, the discrepancy is negative, not positive. The red and green boundary lines are 1/4 of an edostep apart. The vm7 is sometimes called a d7, a v7 or a M7.

Figure 5.3 – The discrepancy of the zo 7th 7/4 and the keyspan of the ru comma 64/63 plotted onto the scale tree



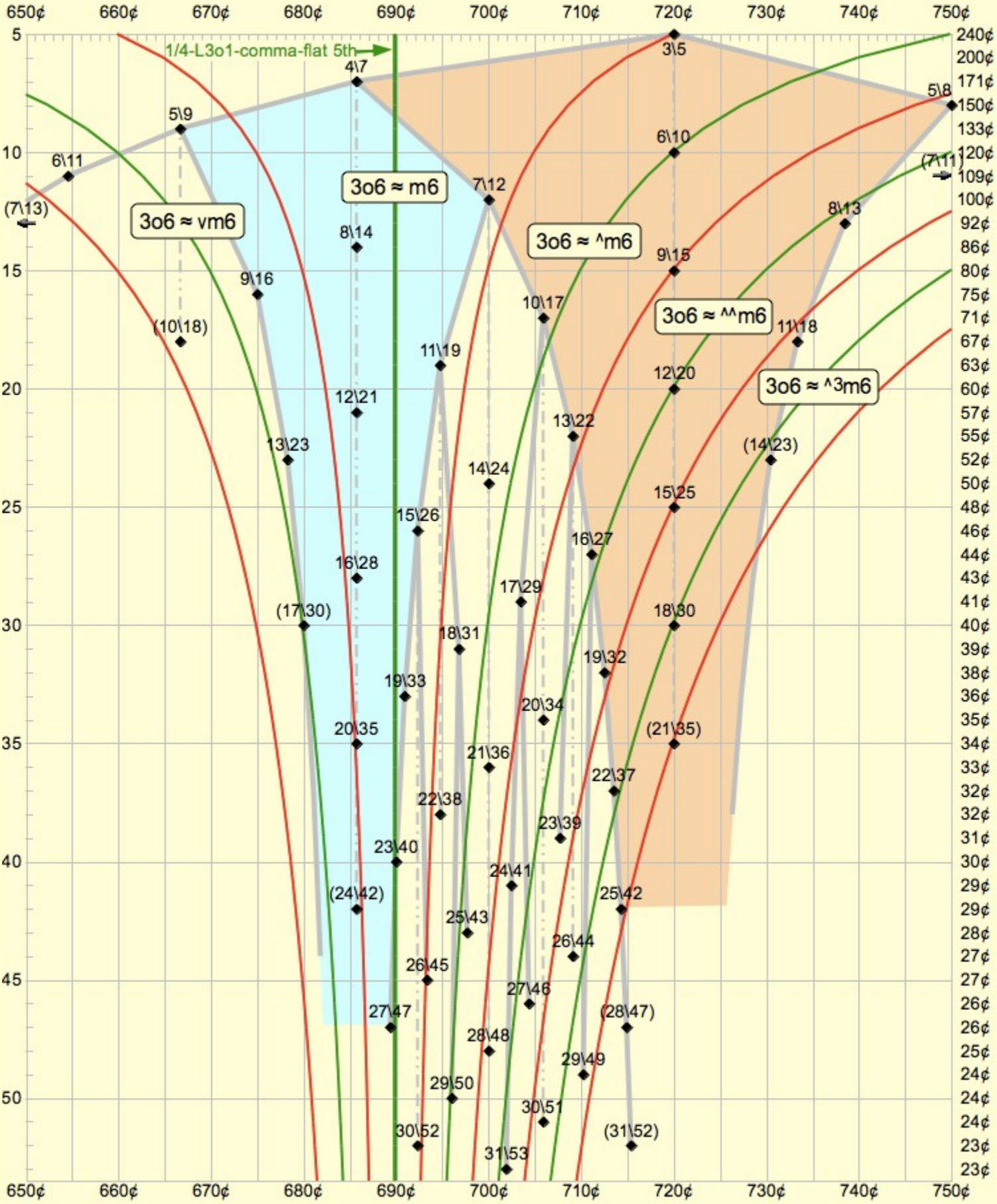
The edo's discrepancy for 11/8 (1o4) exactly equals the distance to the nearest green line. The red and green boundary lines are half an edostep apart. For most mid-sized edos, 11/8 is an ^4, which is sometimes notated as A4 or d4.

Figure 5.4 – The discrepancy of the ilo 4th 11/8 and the keyspan of the ilo comma 33/32



The edo's discrepancy is always some multiple of its distance from the nearest green line. This multiple depends on the comma. The large tho comma 1053/1024 equates 13/8 (3o6) to four 4ths, so the multiplying factor is four, and the straight line is a fourth-comma flat 5th. The red and green lines are one eighth (half of one fourth) of an edostep apart.

Figure 5.5 – The discrepancy of the tho 6th 13/8 and the keyspan of the large tho comma 1053/1024



Section 6 – Formula To Find the Keyspan of an Edo's 5th

When working with many different edos, it can be hard to remember the keyspan of various intervals. This next table shows a method for easily finding many keyspans. It's based on the fact that the white line in Figure 5.1 tends to go up-up-down-up-down every five edos, especially when 13-edo and 18-edo are tweaked.

Table 6.1 – Keyspan of the major 2nd in edos 5-54 (* asterisk indicates tweaked edos 13b and 18b)

5-edo	1	–	1	–	1	9-edo
10-edo	2	1	2	1*	2	14-edo
15-edo	3	2	3	2*	3	19-edo
20-edo	4	3	4	3	4	24-edo
25-edo	5	4	5	4	5	29-edo
30-edo	6	5	6	5	6	34-edo
35-edo	**5**	6	7	6	7	39-edo
40-edo	**6**	7	8	7	8	44-edo
45-edo	**7**	8	**7**	8	9	49-edo
50-edo	**8**	9	**8**	9	10	54-edo

This table shows the keyspan of the major 2nd in various edos. The major 2nd is defined as the interval between the best approximations of 4/3 and 3/2. Thus for 16-edo, the major 2nd is 2\16 = 150¢, not 3\16 = 225¢, even though the latter is closer to 9/8.

The top row is for edos 5 through 9, the next row is for edos 10 through 14, etc. Supersharp and trivial edos are either tweaked to be superflat or omitted. The table follows a surprisingly regular pattern. For N-edo, the keyspan of M2 is always even when N is even, and odd when N is odd. It's usually equal to N divided by 5, rounded down to either an even or an odd number, as needed. Thus M2 = roundDown (N / 5). Exceptions are shown in red.

The M2 keyspan can be used to find other keyspans. For example, for 22-edo:

M2 = 22 / 5 rounded down = 4.4 rounded down = 4 edosteps
P5 = half of M9 = half of (P8 + M2) = (22 + 4) / 2 = 13 edosteps
P4 = P5 - M2 = 13 - 4 = 9 edosteps
m2 = P4 - 2 M2 = 9 - 2·4 = 1 edostep
A1 = M2 - m2 = 4 - 1 = 3 edosteps, hence 22-edo is sharp-3

For 31-edo, since 31 is odd, M2's keyspan must be odd as well.

M2 = 31 / 5 rounded down = 6.2 rounded down to an odd number = 5 edosteps
P5 = (31 + 5) / 2 = 18 edosteps
P4 = 18 - 5 = 13 edosteps
m2 = 13 - 2·5 = 3 edosteps
A1 = 5 - 3 = 2 edosteps, hence 31-edo is sharp-2

More intervals: m7 = P8 - M2 = 31 - 5 = 26 edosteps, and m3 = P4 - M2 = 13 - 5 = 8 edosteps.

This method gives the wrong answer for some edos. The first column errs for those edos which are multiples of 5, but not pentatonic, 35-edo and up. On the scale tree, this is where the spine on the pentatonic kite ends. The third column errs where the lefthand edge of the pentatonic kite ends, 47-edo and up. The fifth column errs from 64-edo on. Above 60-edo, this formula is wrong more often than right, and isn't recommended. But it works for most midsized edos.

Appendix – A Brief Guide to Color Notation

Colors are used when discussing JI considerations of edo intervals and chords.

prime	written	spoken	color	example
3-all	w	wa	white	3/2 = w5 = wa 5th
5-over	y	yo	yellow	5/4 = y3 = yo 3rd
5-under	g	gu	green	6/5 = g3 = gu 3rd
7-over	z	zo	blue/azure	7/6 = z3 = zo 3rd
7-under	r	ru	red	9/7 = r3 = ru 3rd
11-over	1o	lo/ilo	——	11/8 = 1o4 = ilo 4th
11-under	1u	lu	——	12/11 = 1u2 = lu 2nd
13-over	3o	tho	——	13/8 = 3o6 = tho 6th
13-under	3u	thu	——	16/13 = 3u3 = thu 3rd

Over means the prime is in the numerator, under means the denominator, and all means either or neither. The 7-limit colors use a rainbow analogy, with warm colors being sharp and cool colors being flat:

rainbow	temperature	pitch	quality	prime	example
red	hot	sharpest	supermajor	7-under	9/7
yellow	warm	sharper	major	5-over	5/4
green	cool	flatter	minor	5-under	6/5
blue	cold	flattest	subminor	7-over	7/6

Colors can be combined: 7/5 is a zogu 5th. The written and spoken forms for primes 11 and 13 are derived directly from the words eleven and thirteen. Ilo is an alternate form of lo, to avoid confusion with e.g. "low C". Ilo isn't used when lo appears with other syllables; 11/7 is loru not iloru.

For a full explanation of color notation, see the notation guide for JI.