pergen (/pergen)

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Definition

A **pergen** (pronounced "peer-gen") is a way of identifying a regular temperament solely by its period and generator(s). For any temperament, there are many possible periods and generators. For the pergen, they are chosen to use the fewest, and smallest, prime factors possible. Fractions are allowed, e.g. half-octave, but avoided if possible.

If a rank-2 temperament uses the primes 2 and 3 in its comma(s), or in its prime subgroup (i.e. doesn't explicitly exclude the octave or the fifth), then the period can be expressed as the octave 2/1, or some fraction of an octave. Furthermore, the generator can usually be expressed as some 3-limit interval, or some fraction of such an interval. Both fractions are always of the form 1/N, thus the octave and/or the 3-limit interval is **split** into N parts. The interval which is split into multiple generators is the **multigen**. The 3-limit multigen is referred to not by its ratio but by its conventional name, e.g. P5, M6, m7, etc.

For example, the srutal temperament (2.3.5 and 2048/2025) splits the octave in two, and its spoken pergen name is half-octave. The pergen is written (P8/2, P5). Not only the temperament, but also the comma is said to split the octave. The dicot temperament (2.3.5 and 25/24) splits the fifth in two, and is called half-fifth, written (P8, P5/2). Porcupine is third-fourth, or perhaps third-of-a-fourth, (P8, P4/3). Semaphore, a pun on "semi-fourth", is of course half-fourth.

Many temperaments share the same pergen. This has the advantage of reducing the thousands of temperament names to a few dozen categories. It focuses on the melodic properties of the temperament, not the harmonic properties. MOS scales in both srutal and injera sound the same, although they temper out different commas. In addition, the pergen tells us how to notate the temperament using ups and downs. See the notation guide below, under Supplemental materials.

The largest category contains all single-comma temperaments with a comma of the form $2^x 3^y P$ or $2^x 3^y P^{-1}$, where P is a prime > 3 (a **higher prime**), e.g. 81/80 or 135/128. It also includes all commas in which the higher-prime exponents are setwise coprime. The period is the octave, and the generator is the fifth: (P8, P5). Such temperaments are called **unsplit**.

Every temperament has at least one alternate generator, and more, if the octave is split. To avoid ambiguity, the generator is chosen to minimize the amount of splitting of the multigen, and as a tie-breaker, to minimize the size in cents of the multigen. There is only one exception to this rule: the fifth is preferred over the fourth, to follow historical precedent.

For example, srutal could be (P8/2, M2/2), but P5 is preferred because it is unsplit. Or it could be (P8/2, P12), but P5 is preferred because it is smaller. Or it could be (P8/2, P4), but P5 is always preferred over P4. Note that P5/2 is not preferred over P4/2. For example, decimal is (P8/2, P4/2), not (P8/2, P5/2).

pergen		example temperaments			
written	spoken	comma(s)	name	color name	
(P8, P5)	unsplit	81/80	meantone	green	gT
"	"	64/63	archy	red	rT
"	"	(-14,8,1)	schismic	large yellow	LyT
(P8/2, P5)	half-8ve	(11, -4, -2)	srutal	small deep green	sggT
"	"	81/80, 50/49	injera	deep reddish and green	rryy&gT
(P8, P5/2)	half-5th	25/24	dicot	deep yellow	yyT
"	"	(-1,5,0,0,-2)	mohajira	deep amber	aaT
(P8, P4/2)	half-4th	49/48	semaphore	deep blue	bbT
(P8/2, P4/2)	half-everything	25/24, 49/48	decimal	deep yellow and deep blue	yy&bbT
(P8, P4/3)	third-4th	250/243	porcupine	triple yellow	y3T
(P8, P11/3)	third-11th	(12,-1,0,0,-3)	small triple amber	small triple amber	sa3T
(P8/4, P5)	quarter-8ve	(3,4,-4)	diminished	quadruple green	g4T
(P8/2, M2/4)	half-8ve quarter-tone	(-17,2,0,0,4)	large quadruple jade	large quadruple jade	Lj4T
(P8, P12/5)	fifth-12th	(-10,-1,5)	magic	large quintuple yellow	Ly5T

(P8/2, P4/2) is called half-everything because the fifth is also split in half, and since every 3-limit interval can be expressed as the sum/difference of octaves and fifths, every single 3-limit interval is also split in half.

The multigen is usually some voicing of the 4th or 5th, but can be any 3-limit interval, as in the second to last example. The color name indicates the amount of splitting: deep (double) splits something into two parts, triple into three parts, etc. For a comma with monzo (a,b,c,d...), the **color depth** is GCD (c,d...).

Rank-3 pergens have three intervals, period, gen1 and gen2, any or all of which may be split. The pergen always uses at least one higher prime. Color notation (or any HEWM notation) can be used to indicate higher primes. Monzos of the form (a,b,1) = yellow, (a,b,-1) = green, (a,b,0,1) = blue, (a,b,0,-1) = red, (a,b,0,0,1) = jade, (a,b,0,0,-1) = amber, and (a,b) = white. Examples: 5/4 = y3 = yellow 3rd, 7/5 = bg5 = blue-green 5th or bluish 5th, etc.

For example, marvel (2.3.5.7 and 225/224) has an unsplit pergen of (P8, P5, y3). But colors can be replaced with ups and downs, to be higher-prime-agnostic. Here, gen2 can be reduced to g1 = 81/80. Since 81/80 maps to a perfect unison, it can be notated by an up symbol, and we have (P8, P5, ^1) = rank-3 unsplit.

More examples: Triple bluish (2.3.5.7 with 1029/1000 tempered out) is (P8, P11/3, ^1) = rank-3 third-11th. Deep reddish (2.3.5.7 and 50/49) is (P8/2, P5, ^1) = rank-3 half-8ve (^1 = 81/80). However, deep reddish minus white (2.5.7 and 50/49) is (P8/2, M3) = half-8ve, major 3rd.

A rank-4 temperament has a pergen of four intervals, rank-5 has five intervals, etc. A rank-1 temperament could have a pergen of one, such as (P8/12) for 12-edo or (P12/13) for 13-ed3, but there's no particular reason to do so. In fact, edos and edonois are simply rank-1 pergens, and what the concept of edos or edonois does for rank-1 temperaments, the concept of pergens does for temperaments of rank 2 or higher.

In keeping with the higher-prime-agnostic nature of pergens, untempered just intonation has a pergen of the octave, the fifth, and a list of commas, each containing only one higher prime. The higher prime's exponent in the comma's monzo must be ±1, i.e. the color depth must be 1. Furthermore, the comma should map to P1, e.g. 81/80 or 64/63. The commas are notated with microtonal accidentals: (P8, P5, ^1, /1,...).

Derivation

For any comma, let m = the GCD of all the monzo's exponents other than the 2-exponent, and let n = the GCD of all its higher-prime exponents, where GCD (0,x) = x. The comma will split the octave into m parts, and if n > m, it will split some 3-limit interval into n parts.

In a multi-comma rank-2 or higher temperament, it's possible that one comma will contain only the 1st and 2nd primes, and the 2nd prime is directly related to the 1st prime, i.e. it is the period or a multiple of it. Such a prime is **dependent** on a lower prime. If this happens, the multigen must use the 1st and 3rd primes. If the 3rd prime is also dependent, the 4th prime is used, and so forth. In other words, the multigen uses the first two **independent** primes.

For example, consider 2.3.5.7 with commas 256/243 and 225/224. The first comma splits the octave into 5 parts, and makes the 5th be exactly 3/5 of the octave. The pergen is (P8/5, ^1), the same as Blackwood (see Blackwood-like pergens below).

To find a temperament's pergen, first find the period-generator mapping. This is a matrix with a column for each prime in the subgroup, and a row for each period/generator. Not all such mappings will work, the matrix must be in row echelon form. Graham Breed's website has a temperament finder x31eq.com/temper/uv.html that will find such a matrix, it's the reduced mapping. Next make a **square mapping** by discarding columns, usually the columns for the highest primes. But lower primes that are dependent need to be discarded, as in the previous (P8/5, ^1) example, to ensure that the diagonal has no zeros. Lower primes may also be discarded to minimize splitting, see the Breedsmic example below. Then invert the matrix to get the monzos for each period/generator. Add/subtract periods from the generator to get alternate generators. If the interval becomes descending, invert it. For rank-3, add/subtract both periods and generators from the 2nd generator to get more alternates. Choose among the alternates to minimize the splitting and the cents.

For rank-2, we can compute the pergen directly from the square matrix = [(x y), (0, z)]. Let the pergen be (P8/m, M/n), where M is the multigen, P is the period P8/m, and G is the generator M/n.
2/1 = P8 = x·P, thus P = P8/x
3/1 = P12 = y·P + z·G, thus G = [P12 - y·(P8/x)] / z = [-y·P8 + x·P12] / xz = (-y, x) / xz

M's 3-limit monzo is (-y, x), or (y, -x) if z is negative. To get alternate generators, add i periods to G, with i ranging from -x (subtracting a full octave) to +x (adding a full octave).
G' = G + i·P = (-y, x) / xz + i·P8/x = (i·z - y, x) / xz

The rank-2 pergen from the [(x, y), (0, z)] square mapping is (P8/x, (i·z-y,x)/xz), with |i| <= x

A corollary of this formula is that if the octave is unsplit, the multigen is some voicing of the fifth, i.e. some perfect interval. Imperfect multigens like M2 or m3 are fairly rare. Less than 4% of all pergens have an imperfect multigen.

For example, porcupine (2.3.5 and 250/243) has a mapping [(1, 2, 3) (0, -3, -5)] and a square mapping of [(1, 2) (0, -3)]. The pergen is (P8/1, (-3i-2,1)/(-3)) = (P8, (3i+2,-1)/3), with -1 <= i <= 1. No value of i reduces the fraction, so the best multigen is the one with the least cents, (2,-1) = P4. The pergen is (P8, P4/3).

Rank-3 pergens are trickier to find. For example, Breedsmic is 2.3.5.7 with 2401/2400 = (-5,-1,-2,4) tempered out. x31.com gives us this matrix:

	2/1	3/1	5/1	7/1
period	1	1	1	2
gen1	0	2	1	1
gen2	0	0	2	1

Thus 2/1 = P, 3/1 = P + 2·G1, 5/1 = P + G1 + 2·G2, and 7/1 = 2·P + G1 + G2. Discard the last column to make a square matrix with zeros below the diagonal, and no zeros on the diagonal:

	2/1	3/1	5/1
period	1	1	1
gen1	0	2	1
gen2	0	0	2

Use an online tool to invert it. Here "/4" means that each entry is to be divided by the determinant of the last matrix, which is 4.

	period	gen1	gen2	
2/1	4	-2	-1	
3/1	0	2	-1	
5/1	0	0	2	/4

Thus the period = (4,0,0) / 4 = 2/1= P8, gen1 = (-2,2,0) / 4 = (-1,1,0) / 2 = P5/2, and gen2 = (-1,-1,2) / 4 = (25:6)/4.

Next, search for alternate generators. Add/subtract the period 2/1 from gen1. Since the multigen P5 is split in half, one multigen equals two gens, and adding an octave to the gen adds a double octave to the multigen. The alternate gens are P11/2 and P19/2, both of which are much larger, so the best gen1 is P5/2.

The 2nd multigen is split into quarters, so we must add/subtract quadruple periods and generators to it. Subtracting a quadruple octave and inverting makes gen2 be (5,1,-2)/4 = (96:25)/4. The multigen is a diminished double octave. A quadruple half-5th is a double 5th is a M9. Subtracting that makes gen2 be (128:75)/4, quarter-dim-7th. Subtracting M9 again, and inverting again, makes gen2 = (-9,3,2)/4 = (675:512)/4, quarter-aug-3rd. As gen2's cents become smaller, the odd limit becomes greater, the intervals remain obscure, and the notation remains awkward.

Fortunately, there is a better way. Discard the 3rd column instead, and keep the 4th one:

	2/1	3/1	7/1
period	1	1	2
gen1	0	2	1
gen2	0	0	1

This inverts to this matrix:

	period	gen1	gen2	
2/1	2	-1	-3	
3/1	0	1	-1	
7/1	0	0	2	/2

Again, period = P8 and gen1 = P5/2. Gen2 = (-3,-1,2)/2. To add gen1 to gen2, add a double gen1 to the 2nd multigen. A double half-5th is a 5th = (-1,1,0), and this gives us (-4,0,2)/2 = (-2,0,1) = 7/4. The fraction disappears, the multigen becomes the gen, and we can add/subtract the period and the gen1 directly. Subtracting an octave and inverting makes gen2 = 8/7. Adding an octave and subtracting 4 half-5ths makes 64/63. The pergen is (P8, P5/2, /1) with /1 = 64/63, rank-3 half-5th. This is far better than (P8, P5/2, (96:25)/4).

The pergen sometimes uses a larger prime in place of a smaller one in order to avoid splitting gen2, but only if the smaller prime is > 3. In other words, the first priority is to have as few higher primes (colors) as possible, next to have as few fractions as possible, finally to have the higher primes be as small as possible.

Applications

Pergens allow for a systematic exploration of all posible rank-2 tunings, potentially identifying new musical resources.

Another obvious application is to name regular temperaments in a logical, consistent manner, avoiding the need to memorize many arbitrary names. Many temperaments have pergen-like names: Hemififths is (P8, P5/2), semihemi is (P8/2, P4/2), triforce is (P8/3, P4/2), both tetracot and semihemififths are (P8, P5/4), fourfives is (P8/4, P5/5), pental is (P8/5, P5), and fifive is (P8/2, P5/5). Pergen names are an improvement over these because they specify more exactly what is split. Some temperament names are what might be called a pseudo-pergen, either because it contains more than 2 primes, or because the split multigen isn't actually a generator. For example, sensei, or semisixth, implies a pseudo-pergen (P8, (5/3)/2) that contains 3 primes. Meantone (mean = average, tone = major 2nd) implies a pseudo-pergen (P8, (5/4)/2), only 2 primes, but the tone isn't a generator.

Pergens group many temperaments into one category, which has its advantages and its disadvantages. Some temperament names also do this, for example porcupine refers to not only 2.3.5 with 250/243, but also 2.3.5.7 with 250/243 and 64/63. Color names are the only type of name that never does this. The first porcupine is triple yellow, and the second one is triple yellow and red. Together, the pergen name and the color name supply a lot of information. The pergen name indicates the melodic possibilities in a higher-primes-agnostic manner, and the color name indicates the harmonic possibilities: the prime subgroup, and what types of chord progressions it supports. Both names indicate the rank, the pergen name more directly.

Pergens can also be used to notate rank-2 scales, e.g. (P8, P4/3) [7] = third-4th heptatonic is a higher-primes-agnostic name for the Porcupine [7] scale. As long as the 5th is tuned fairly accurately, any two temperaments that have the same pergen tend to have the same MOS scales. See Chord names and scale names below.

The final main application, which the rest of this article will focus on, is that pergens allow a systematic approach to notating regular temperaments, without having to examine each of the thousands of individual temperaments. The discussion mostly focuses on rank-2 temperaments that include primes 2 and 3.

All unsplit temperaments can be notated identically. They require only conventional notation: 7 nominals, plus sharps and flats. All other rank-2 temperaments require an additional pair of accidentals, ups and downs. Certain rank-2 temperaments require another additional pair, **highs and lows**, written / and \. Dv\ is down-low D, and /5 is a high-fifth. Alternatively, color accidentals (y, g, r, b, j, a, etc.) could be used. However, this constrains a pergen to a specific temperament. For example, both mohajira and dicot are (P8, P5/2). Using y and g implies dicot, using j and a implies mohajira, but using ^ and v implies neither, and is a more general notation.

One can avoid additional accidentals for all rank-1 and rank-2 tunings (but not rank-3 or higher ones) by sacrificing backwards compatibility with conventional notation, which is octave-equivalent, fifth-generated and heptatonic. Porcupine can be notated without ups and downs if the notation is 2nd-generated. Half-octave can be notated decatonically. However, one would sacrifice the interval arithmetic and staff notation one has spent years internalizing, and naming chords becomes impossible. The sacrifice is too great to take lightly, and all notation used here is backwards compatible.

Analogous to 22-edo, sometimes additional accidentals aren't needed, but are desirable, to avoid misspelled chords. For example, schismic is unsplit and can be notated conventionally. But this causes 4:5:6 to be spelled not as stacked thirds but as C Fb G. With ^1 = 81/80, the chord can be spelled properly as C Ev G. See Notating unsplit pergens below.

Not all combinations of periods and generators are valid. Some are duplicates of other pergens. (P8/2, M2/2) is actually (P8/2, P5). Some combinations are impossible. There is no (P8, M2/2), because no combination of periods and generators equals P5.

Below is a table that lists all the rank-2 pergens that contain primes 2 and 3, up to third-splits. They are grouped into blocks by the size of the larger splitting fraction, and grouped within each block into sections by the smaller fraction. Most sections have two halves. In the first half, the octave has the larger fraction, in the second, the multigen does. Within each half, the pergens are sorted by multigen size. This is a convenient lexicographical ordering of rank-2 pergens that enables one to easily look up a pergen in a notation guide. It even allows every pergen to be numbered.

The enharmonic interval, or more briefly the **enharmonic**, can be added to or subtracted from any note (or interval), renaming it, but not changing the pitch of the note (or width of the interval). It's analogous to the dim 2nd in 12-edo, which equates C# with Db, and A4 with d5. In a single-comma temperament, the comma usually maps to the pergen's enharmonic. The pergen and the enharmonic together define the notation.

The **genchain** (chain of generators) in the table is only a short section of the full genchain.

C - G implies ...Eb Bb F C G D A E B F# C#...

C - Eb^=Ev - G implies ...F -- Ab^=Av -- C -- Eb^=Ev -- G -- Bb^=Bv -- D...

If the octave is split, the table has a **perchain** ("peer-chain", chain of periods) that shows the octave: C -- F#v=Gb^ -- C. Genchains have a theoretically infinite length, but perchains have a finite length. The full rank-2 lattice has genchains running horizontally and perchains running vertically.

	pergen	enharmonic interval(s)	equivalence(s)	split interval(s)	perchain(s) and/or genchains(s)	examples
1	(P8, P5) unsplit	none	none	none	C - G	pythagorean, meantone, dominant, schismic, mavila, archy, etc.

	half-splits					
2	(P8/2, P5) half-8ve	^^d2 (if 5th > 700¢	C^^ = B#	P8/2 = vA4 = ^d5	C - F#v=Gb^ - C	srutal ^1 = 81/80
	"	vvd2 (if 5th < 700¢)	C^^ = Db	P8/2 = ^A4 = vd5	C - F#^=Gbv - C	injera ^1 = 64/63
	"	vvM2	C^^ = D	P8/2 = ^4 = vP5	C - F^=Gv - C	deep emerald, if 13/8 = M6 ^1 = 27/26
3	(P8, P4/2) half-4th	vvm2	C^^ = Db	P4/2 = ^M2 = vm3	C - D^=Ebv - F	semaphore ^1 = 64/63
	"	^^dd2	C^^ = B##	P4/2 = vA2 = ^d3	C - D#v=Ebb^ - F	double large deep yellow ^1 = 81/80
4	(P8, P5/2) half-5th	vvA1	C^^ = C#	P5/2 = ^m3 = vM3	C - Eb^=Ev - G	mohajira ^1 = 33/32
5	(P8/2, P4/2) half- everything	\\m2, vvA1, ^^\\d2, vv\\M2	C// = Db C^^ = C# C^^// = D	P4/2 = /M2 = \m3 P5/2 = ^m3 = vM3 P8/2 = v/A4 = ^\d5 = ^/4 = v\P5	C - D/=Eb\ - F, C - Eb^=Ev - G, C - F#v/=Gb^\ - C, C - F^/=Gv\ - C	deep blue & deep amber ^1 = 33/32 /1 = 64/63
	"	^^d2, \\m2, vv\\A1	C^^ = B# C// = Db C^^// = C#	P8/2 = vA4 = ^d5 P4/2 = /M2 = \m3 P5/2 = ^/m3 = v\M3	C - F#v=Gb^ - C, C - D/=Eb\ - F, C - Eb^/=Ev\ - G	small deep green & deep blue ^1 = 81/80 /1 = 64/63
	"	^^d2, \\A1, ^^\\m2	C^^ = B# C// = C# C^^\\ = B	P8/2 = vA4 = ^d5 P5/2 = /m3 = \M3 P4/2 =v/M2 = ^\m3	C - F#v=Gb^ - C, C - Eb/=E\ - G, C - Dv/=Eb^\ - F	small deep green and deep amber ^1 = 81/80 /1 = 33/32
	third-splits					
6	(P8/3, P5) third-8ve	^3d2	C^3 = B#	P8/3 = vM3 = ^^d4	C - Ev - Ab^ - C	augmented ^1 = 81/80
7	(P8, P4/3) third-4th	v3A1	C^3 = C#	P4/3 = vM2 = ^^m2	C - Dv - Eb^ - F	porcupine ^1 = 81/80
8	(P8, P5/3) third-5th	v3m2	C^3 = Db	P5/3 = ^M2 = vvm3	C - D^ - Fv - G	slendric ^1 = 64/63
9	(P8, P11/3) third-11th	^3dd2	C^3 = B##	P11/3 = vA4 = ^^dd5	C - F#v - Cb^ - F	small triple amber, if 11/8 = A4 ^1 = 729/704
	"	v3M2	C^3 = D	P11/3 = ^4 = vv5	C - F^ - Cv - F	small triple amber, if 11/8 = P4 ^1 = 33/32
10	(P8/3, P4/2) third-8ve, half-4th	v6A2	C^6 = D#	P8/3 = ^^m3 = v4A4 P4/2 = ^3m2 = v3M3	C - Eb^^ - Avv - C C - Db^3=Ev3 - F	sixfold jade, if 11/8 = P4 ^1 = 33/32
	"	^3d2, \\m2	C^3 = B# C// = Db	P8/3 = vM3 = ^^d4 P4/2 = /M2 = \m3	C - Ev - Ab^ - C C - D/=Eb\ - F	triforce (128/125 & 49/48) ^1 = 81/80, /1 = 64/63
11	(P8/3, P5/2) third-8ve, half-5th	^3d2 \\A1	C^3 = B# C// = C#	P8/3 = vM3 = ^^d4 P5/2 = /m3 = \M3	C - Ev - Ab^ - C C - Eb/=E\ - G	small sixfold blue ^1 = 49/48, /1 = 343/324
12	(P8/2, P4/3) half-8ve, third-4th	^^d2 \3A1	C^^ = Dbb C/3 = C#	P8/2 = vA4 = ^d5 P4/3 = \M2 = //m2	C - F#v=Gb^ - C C - D\ - Eb/ - F	large sixfold red ^1 = 1029/1024, /1 = 49/48
13	(P8/2, P5/3) half-8ve, third-5th	^6d32	C^6 = B#3	P8/2 = v3AA4 = ^3dd5 P5/3 = vvA2 = ^4dd3	C - Fxv3=Gbb^3 C C - D#vv - Fb^^ - G	large sixfold yellow ^1 = 81/80
	"	^^d2, \\\m2	C^^ = B# C/// = Db	P8/2 = vA4 = ^d5 P5/3 = /M2 = \\m3	C - F#v=Gb^ - C C - D/ - F\ - G	lemba (50/49 & 1029/1024) ^1 = (10,-6,1,-1), /1 = 64/63
14	(P8/2, P11/3) half-8ve, third-11th	v6M2	C^6 = D	P8/2 = ^34 = v35 P11/3 = ^^4 = v45	C - F^3=Gv3 - C C - F^^ - Cvv - F	large sixfold jade, if 11/8 = P4 ^1 = 33/32
15	(P8/3, P4/3) third- everything	v3d2, \3A1	C^3 = Dbb C/3 = C#	P8/3 = ^M3 = vvd4 P4/3 = \M2 = //m2 P5/3 = v/M2	C - E^ - Abv - C C - D\ - Eb/ - F C - Dv/ - F^\ - G	250/243 & 729/686 ^1 = 64/63 /1 = 81/80
	"	^3d2, \3m2	C^3 = B# C/3 = Db	P8/3 = vM3 = ^^d4 P5/3 = /M2 = \\m3 P4/3 = v\M2	C - Ev - Ab^ - C C - D/ - F\ - G C - Dv\ - Eb^/ - F	triple green & large triple blue ^1 = 81/80 /1 = 64/63
	"	v3A1, \3m2	C^3 = C# C/3 = Db	P4/3 = vM2 = ^^m2 P5/3 = /M2 = \\m3 P8/3 = v/M3	C - Dv - Eb^ - F C - D/ - F\ - G C - Ev/ - Ab^\ - C	triple yellow & large triple blue ^1 = 81/80 /1 = 64/63
	quarter-splits					
16	(P8/4, P5)	^4d2	C^4 = B#	P8/4 = vm3 = ^3A2	C Ebv Gbvv=F#^^ A^ C	diminished
17	(P8, P4/4)	^4dd2	C^4 = B##	P4/4 = ^m2 = v3AA1	C Db^ Ebb^^=D#vv Ev F	negri
18	(P8, P5/4)	v4A1	C^4 = C#	P5/4 = vM2 = ^3m2	C Dv Evv=Eb^^ F^ G	tetracot
19	(P8, P11/4)	v4dd3	C^4 = Eb3	P11/4 = ^M3 = v3dd5	C E^ G#^^ Dbv F	squares
20	(P8, P12/4)	v4m2	C^4 = Db	P12/4 = v4 = ^3M3	C Fv Bbvv=A^^ D^ G	vulture
	etc.					

The disadvantage to the lexicographical ordering above is that more complex pergens are listed before simpler ones, e.g. half-8ve third-5th before quarter-5th. However, the former can arise from two simple commas, so arguably it isn't all that complex.

Some pergens are not very musically useful. (P8/2, P11/3) has a period of about 600¢ and a generator of about 566¢, or equivalently 34¢. The generator is much smaller than the period, and MOS scales will have a very lopsided L/s ratio. (P8/3, P5/2) is almost as lopsided (P = 400¢, G = 50¢).

Tipping points

Removing the ups and downs from an enharmonic interval makes a "bare" enharmonic, a conventional 3-limit interval which vanishes in certain edos. For example, (P8/2, P5)'s enharmonic interval is ^^d2, the bare enharmonic is d2, and d2 vanishes in 12-edo. Every rank-2 temperament has a "sweet spot" for tuning the 5th, usually a narrow range of about 5-10¢. 12-edo's fifth is the "tipping point": if the temperament's 5th is flatter than 12-edo's, d2 is ascending, and if it's sharper, it's descending. The ups and downs are meant to indicate that the enharmonic interval vanishes. Thus if d2 is ascending, it should be downed, and if it's descending, upped. Therefore **up may need to be swapped with down, depending on the size of the 5th** in the particular rank-2 tuning you are using. In the above table, this is shown explicitly for (P8/2, P5), and implied for all the other pergens. In the table, the other pergens' enharmonic intervals are upped or downed as if the 5th were just.

Heptatonic 5th-based notation is only possible if the 5th ranges from 600¢ to 720¢. For every bare enharmonic, the following table shows in what parts of this range this interval should be upped or downed. The tipping point edo is simply the 3-exponent of the bare enharmonic.

bare enharmonic interval		3-exponent	tipping point edo	edo's 5th	upping range	downing range	if the 5th is just
M2	C - D	2	2-edo	600¢	none	all	downed
m3	C - Eb	-3	3-edo	800¢	none	all	downed
m2	C - Db	-5	5-edo	720¢	none	all	downed
A1	C - C#	7	7-edo	~686¢	600-686¢	686¢-720¢	downed
d2	C - Dbb	-12	12-edo	700¢	700-720¢	600-700¢	upped
dd3	C - Eb3	-17	17-edo	~706¢	706-720¢	600-706¢	downed
dd2	C - Db3	-19	19-edo	~695¢	695-720¢	600-695¢	upped
d34	C - Fb3	-22	22-edo	~709¢	709-720¢	600-709¢	downed
d32	C - Db4	-26	26-edo	~692¢	692-720¢	600-692¢	upped
d44	C - Fb4	-29	29-edo	~703¢	703-720¢	600-703¢	downed
d43	C - Eb5	-31	31-edo	~697¢	697-720¢	600-697¢	upped
etc.							

Further Discussion

Naming very large intervals

So far, the largest multigen has been a 12th. As the multigen fractions get bigger, the multigen can get quite large. To avoid cumbersome degree names like 16th or 22nd, for degrees above 12, **widening** by an 8ve is indicated by "W". Thus 10/3 = WM6 = wide major 6th, 9/2 = WWM2 or WM9, etc. For a pergen with an unsplit octave, the multigen is some voicing of the fifth that is less than n/2 octaves. For (P8, M/6), the multigen M is less than 3 octaves, and can be P4, P5, P11, P12, WWP4 or WWP5.

Secondary splits

Besides the octave and/or multigen, a pergen splits many other 3-limit intervals as well. The composer can use these secondary splits to create melodies with equal-sized steps. For example, third-4th (e.g. porcupine) splits intervals other than the 4th into three parts. Of course, many 3-limit intervals split into three parts even when untempered, e.g. A4 = 3·M2. The interval's monzo (a,b) must have both a and b divisible by 3. The third-4th pergen furthermore splits any interval which is the sum or difference of the 4th and the triple 8ve WWP8. Stacking 4ths gives these intervals: P4, m7, m10, Wm6, Wm9... The last two are too large to be of any use melodically. Subtracting 4ths from the triple 8ve gives WWP8, WWP5, WM9, WM6, M10, M7, A4... The first four are too large, this leaves us with:

P4/3: C - Dv - Eb^ - F
A4:/3 C - D - E - F# (the lack of ups and downs indicates that this interval was already split)
m7/3: C - Eb^ - Gv - Bb (also m7/6: C - Dv - Eb^ - F - Gv - Ab^ - Bb)
M7/3: C - Ev - G^ - B
m10/3: C - F - Bb - Eb (also already split) (m10/9 also occurs)
M10/3: C - F^ - Bv - E

Of course, an equal-step melody can span other intervals besides 3-limit ones. Simply extend or cut short the earlier examples to find other melodies:

^m3/2: C - Dv - Eb^ (^m3 = 6/5)
^m6/5: C - Dv - Eb^ - F - Gv - Ab^ (^m6 = 8/5)
vm9/4: C - Eb^ - Gv - Bb - Db^ (vm9 = 32/15)
vM7/2: C - F^ - Bv (vM7 = 15/8, probably more harmonious than M7 = 243/128)

More remote intervals include A1, d4, d7 and d10. These require a very long genchain. The most interesting melodically is A1: C - C^ - C#v - C#. From C to C# is 7 5ths, which equals 21 generators, so the genchain would contain 22 notes if it had no gaps.

For a pergen (P8, (a,b)/n), any interval generated by n octaves and the multigen splits into at least n parts. For a pergen (P8/m, P5), any interval generated by the octave and m 5ths splits into at least m parts. Thus any naturally occurring split of m parts occurs in all voicings of that interval. For example, M9 naturally splits into two 5ths, therefore (P8/2, P5) splits all voicings of M9, including M2.

Given a pergen (P8/m, (a,b)/n), an interval (a',b') splits into GCD ((a'·b - a·b')·m/b, b'·n/b) parts (proof below). For an unsplit pergen, we have the naturally occurring split of GCD (a', b'). If only the 8ve is split, we have GCD (a'·m, b'). If m = n (an nth-everything pergen), we have n·GCD (a',b'). If the enharmonic is an A1, every interval with a degree of n+1 will be split. Thus half-5th splits every 3rd, 5th, 7th , 9th, etc. in half. "Every" means every quality, so 3rd includes d3, m3, M3 and A3, 5th includes P5, A5 and d5, etc.

The following table shows the secondary splits for all pergens up to the third-splits. A split interval is only included if it falls in the range from d5 to A5 on the genchain of 5ths, others are too remote. For convenience, naturally occurring splits are listed too, under "all pergens". (P8/3, P4/2) has all the secondary splits that (P8/3, P5) and (P8, P4/2) have, plus additional ones.

pergen		secondary splits of a 12th or less
all pergens		M3/2, A4/3, d5/2, A5/4, m7/2, M9/2, m10/3, A11/2
half-splits		
(P8/2, P5)	half-8ve	M2/2, M3/4, A4/6, A5/8, m6/2, M10/2, d12/2, A12/2
(P8, P4/2)	half-4th	m2/2, M6/2, m7/4, A8/2, m10/6, A11/4, P12/2
(P8, P5/2)	half-5th	A1/2, m3/2, M7/2, m9/2, P11/2
(P8/2, P4/2)	half-everything	every 3-limit interval is split twice as much as before
third-splits		
(P8/3, P5)	third-8ve	m3/3, M6/3, d5/6, A11/3, d12/3
(P8, P4/3)	third-4th	A1/3, m7/6, M7/3, m10/9, M10/3
(P8, P5/3)	third-5th	m2/3, m6/3, M9/6, A8/3, A12/3
(P8, P11/3)	third-11th	M2/3, M3/6, A4/9, A5/12, m9/3, P12/3
(P8/3, P4/2)	third-8ve half-4th	third-8ve splits, half-4th splits, M6/6, m10/6, A11/12
(P8/3, P5/2)	third-8ve half-5th	third-8ve splits, half-5th splits, m3/6, d5/12
(P8/2, P4/3)	half-8ve third-4th	half-8ve splits, third-4th splits, A4/6, M10/6
(P8/2, P5/3)	half-8ve third-5th	half-8ve splits, third-5th splits, m6/6, M9/6, A12/6
(P8/2, P11/3)	half-8ve third-11th	half-8ve splits, third-11th splits, M2/6, M3/12, A4/18, A5/24
(P83, P4/3)	third-everything	every 3-limit interval is split three times as much as before

Singles and doubles

If a pergen has only one fraction, like (P8/2, P5) or (P8, P4/3), the pergen is a **single-split** pergen. If it has two fractions, it's a **double-split** pergen. A single-split pergen can result from tempering out only a single comma, although it can be created by multiple commas. A single-split pergen can be notated with only ups and downs, called **single-pair** notation because it adds only a single pair of accidentals to conventional notation. **Double-pair** notation uses both ups/downs and highs/lows. In general, single-pair notation is preferred, because it's simpler. However, double-pair notation may be preferred, especially if the enharmonic for single-pair notation is a 3rd or larger, or if single-pair notation requires using quadruple, quintuple, etc. ups and downs.

In this article, for double-pair notation, the period uses ups and downs, and the generator uses highs and lows. But the choice of which pair of accidentals is used for what is arbitrary, and ups/downs could be exchanged with highs/lows.

Every double-split pergen is either a **true double** or a **false double**. A true double, like third-everything (P8/3, P4/3) or half-8ve quarter-4th (P8/2, P4/4), can only arise when at least two commas are tempered out, and requires double pair notation. A false double, like half-8ve quarter-tone (P8/2, M2/4), can arise from a single comma, and can be notated with single pair notation. Thus a false double behaves like a single-split, and is easier to construct and easier to notate. In a false double, the multigen split automatically splits the octave as well: if M2 = 4·G, then P8 = M9 - M2 = 2·P5 - 4·G = (P5 - 2·G) / 2. In general, if a pergen's multigen is (a,b), the octave is split into at least |b| parts.

A pergen (P8/m, (a,b)/n) is a false double if and only if GCD (m,n) = |b|. The next section discusses an alternate test.

Finding an example temperament

To find an example of a temperament with a specific pergen, we must find the comma(s) the temperament tempers out. To construct a comma that creates a single-split pergen, find a ratio for P or G that contains only one higher prime, with color depth of 1 (i.e. exponent of ±1), of appropriate cents to add up to approximately the octave or the multigen. The comma is the difference between the stacked ratios and the larger interval. For example, (P8/4, P5) requires a P of about 300¢. The comma is the difference between 4·P and P8. If P is 6/5, the comma is 4·P - P8 = $(6/5)^4$ ÷ (2/1) = 648/625, the diminished temperament. If P is 7/6, the comma is P8 - 4·P = (2/1) · $(7/6)^{-4}$, the quadruple red temperament. Neither 13/11 nor 32/27 would work for P, too many and too few higher primes respectively.

Another method: if the generator's cents are known, look on the genchain for an interval that approximates a ratio of color depth ±1. Let the interval be I, and the genspan of this interval be x. Then n·x gens = n·I = x·M, where M is the multigen and M/n is the generator. The comma can be found from this equation, if n and x are coprime. For example, suppose (P8, P5/5) has G = 140¢. The genchain is all multiples of 140¢. Looking at the cents, 280¢ is about 7/6. Thus 2G = 7/6, and 10G = 5·(7/6) = 2·P5. Thus 2·P5 - 5·(7/6) = 0G = 0¢, and the comma is (3, 7, 0, -5). If the period is split and the generator isn't, use the perchain instead of the genchain. For example, (P8/7, P5) has a period of 141¢. 2 gens = 343¢, about 11/9.

Thus 7·(11/9) = 2·P8, and the comma is (-2, -14, 0, 0, 7).

If the pergen's notation is known, an even easier method is to simply assume that the up symbol equals a comma that maps to P1, such as 81/80 or 64/63 (see mapping commas in the next section). Thus for (P8/4, P5), if P = vm3, and ^1 = 64/63, P is 32/27 ÷ 64/63 = 7/6. This method is notation-dependent: (P8/2, P5) with P = vA4 and ^1 = 81/80 gives P = 45/32, but if P = ^4, then P = 27/20.

Finding the comma(s) for a double-split pergen is trickier. As previously noted, if a pergen's multigen is (a,b), the octave is split into at least |b| parts. Therefore if a pergen (P8/m, (a,b)/n) has m = |b|, it is **explicitly false**. If so, proceed as if the octave were unsplit: (P8/2, M2/4) requires G ~ 50¢, perhaps 33/32, and the comma is 4·G - M2 = (33/32)^4 / (9/8) = (-17, 2, 0, 0, 4).

If the pergen is not explicitly false, put the pergen in its **unreduced** form, which is always explicitly false if the pergen is a false double. The unreduced form replaces the generator with the difference between the period and the generator: (P8/m, M/n) becomes (P8/m, P8/m - M/n) = (P8/m, (n·P8 - m·M)/nm) = (P8/m, M'/n'). The new multigen M' is the product of the original pergen's outer elements (P8 and n) minus the product of the inner elements (m and M), divided by the product of the fractions (m and n). Invert M' if descending (if P < G), and simplify if m and n aren't coprime. M' will have a larger fraction and/or a larger size in cents, hence the name unreduced.

For example, (P8/3, P5/2) is a false double that isn't explicitly false. Its unreduced generator is (2·P8 - 3·P5) / (3·2) = m3/6, and the unreduced pergen is (P8/3, m3/6). This is explicitly false, thus the comma can be found from m3/6 alone. G' is about 50¢, and the comma is 6·G' - m3. The comma splits both the octave and the fifth.

This suggests an alternate true/false test: if neither the pergen nor the unreduced pergen is explicitly false, the pergen is a true double. For example, (P8/4, P4/2) isn't explicitly false. The unreduced pergen is (P8/4, M2/4), which also isn't explicitly false, thus (P8/4, P4/2) is a true double. It requires two commas, one for each fraction. The two commas must use different higher primes, e.g. 648/625 and 49/48. Thus true doubles require commas of at least 7-limit, whereas false doubles require only 5-limit. To summarize:

- **A double-split pergen is explicitly false if m = |b|, and not explicitly false if m > |b|.**
- **A double-split pergen is a true double if and only if neither it nor its unreduced form is explicitly false.**
- **A double-split pergen is a true double if GCD (m, n) > |b|, and a false double if GCD (m, n) = |b|.**

A false double pergen's temperament can also be constructed from two commas, as if it were a true double. For example, (P8/3, P4/2) results from 128/125 and 49/48, which split the octave and the 4th respectively.

Unreducing replaces the generator with an alternate generator. Any number of periods plus or minus a single generator makes an **alternate** generator. A generator or period plus or minus any number of enharmonics makes an **equivalent** generator or period. An equivalent generator is always the same size in cents, since the enharmonic is always 0¢. An equivalent generator is the same interval, merely notated differently. For example: (P8, P5/2) has generator ^m3 and equivalent generator vM3. Another example, half-8ve (P8/2, P5) has period vA4 and equivalent period ^d5. It has generator P5 and alternate generators P4 and vA1. vA1 is equivalent to ^m2.

Of the two equivalent generators, the preferred generator is always the smaller one (smaller degree, or if the degrees are the same, more diminished). This is because the equivalent generator can be more easily found by adding the enharmonic, rather than subtracting it. For example, ^m3 + vvA1 = vM3 is an easier calculation than vM3 - vvA1 = ^m3. This is particularly true with complex enharmonics like ^^{6}dd2.

There are also alternate enharmonics, see below. For double-pair notation, there are also equivalent enharmonics.

Ratio and cents of the accidentals

The sharp symbol's ratio is always (-11,7) = 2187/2048, by definition. Looking at the table in the Applications section, the up symbol often equals only a few ratios. For most 5-limit temperaments, ^1 = 81/80. For most 2.3.7 temperaments, ^1 = 64/63. Most 11-limit temperaments use either 33/32 or 729/704. These **mapping commas** are used to map higher primes to 3-limit intervals, and are essential for notation. They also determine where a ratio "lands" on a keyboard. By definition they are a P1, and the only intervals that map to P1 are these commas and combinations of them.

If a single-comma temperament uses double-pair notation, neither accidental will equal the mapping comma. A double-comma temperament using double-pair notation may use the difference between two mapping commas, as in lemba, where ^1 equals 64/63 minus 81/80.

Sometimes the mapping comma needs to be inverted. In diminished, which sets 6/5 = P8/4, ^1 = 80/81. in every temperament except those in the meantone family, the 81/80 comma is not tempered out, but it is still tempered, just like every ratio. Occasionally 81/80 is tempered so far that it becomes a descending interval. See also blackwood-like pergens below.

Cents can be assigned to each accidental symbol, even if no specific commas are specified. Let c = the cents of the tuning's 5th from 700¢, the 12edo 5th. Thus P5 = 700¢ + c. From this we can calculate the cents of any 3-limit interval. The sharp always equals A1 = 100¢ + c. Since the enharmonic = 0¢, we can derive the cents of the up symbol. If the enharmonic is vvA1, then vvA1 = 0¢, and ^1 = (A1)/2 = (100¢ + 7c)/2 = 50¢ + 3.5c. If the 5th is 696¢, c = -4 and the up symbol equals 36¢. /1 can be similarly derived from its enharmonic.

In certain edos, the up symbol's cents can be directly related to the sharp's cents. For example, in 15edo, ^ is 1/3 the cents of #. The same can be done for rank-2 pergens if and only if the enharmonic is an A1.

This suggests a simple format for describing the tuning at the top of the score: the name of the tuning, perhaps the cents of the 5th, and the cents of any accidental pairs used, rounded off to the nearest integer. This gives the musician, who may not be well-versed in microtonal theory, basic information for playing the score. Examples:

15-edo: # = 240¢, ^ = 80¢ (^ = 1/3 #)
16-edo: # = -75¢
17-edo: # = 141¢, ^ = 71¢ (^ = 1/2 #)
18b-edo: # = -133¢, ^ = 67¢ (^ = 1/2 #)
19-edo: # = 63¢
21-edo: ^ = 57¢ (if used, # = 0¢)

22-edo: # = 164¢, ^ = 55¢ (^ = 1/3 #)

quarter-comma meantone: # = 76¢
fifth-comma meantone: # = 84¢
third-comma archy: # = 177¢
eighth-comma porcupine: # = 157¢, ^ = 52¢ (^ = 1/3 #)
sixth-comma srutal: # = 139¢, ^ = 33¢ (no fixed relationship between ^ and #)
third-comma injera: # = 63¢, ^ = 31¢ (no fixed relationship between ^ and #, third-comma means 1/3 of 81/80)
eighth-comma hedgehog: # = 157¢, ^ = 49¢, / = 52¢ (/ = 1/3 #, eighth-comma means 1/8 of 250/243)

Hedgehog is half-8ve third-4th. While the best tuning of a specific temperament is found by minimizing the mistuning of certain ratios, the best tuning of a general pergen is less obvious. Both srutal and injera are half-8ve, but their optimal tunings are very different.

Finding a notation for a pergen

There are multiple notations for a given pergen, depending on the enharmonic interval(s). Preferably, the enharmonic's degree will be a unison or a 2nd, because equating two notes a 3rd or 4th apart is very disconcerting. If it's a unison, it will always be an A1. (P1 would be pointless, d1 would be inverted to A1, and AA1 would be split into two A1's.) If it's a 2nd, preferably it will be a m2 or a d2 or a dd2, and not a M2 or an A2 or a ddd2. There is an easy method for finding such a pergen, if one exists. First, some terminology and basic concepts:

- For (P8/m, M/n), P8 = mP + xE and M = nG + yE', with 0 < |x| <= m/2 and 0 < |y| <= n/2
- x is the count for E, with E occurring x times in one octave, and xE is the octave's **multi-enharmonic**, or **multi-E** for short
- y is the count for E', with E' occurring y times in one multigen, and yE' is the multigen's multi-E
- For false doubles using single-pair notation, E = E', but x and y are usually different, making different multi-enharmonics
- The unreduced pergen is (P8/m, M'/n'), with a new enharmonic E" and new counts, P8 = mP + x'E", and M' = n'G' + y'E"

The **keyspan** of an interval is the number of keys or frets or semitones that the interval spans in 12-edo. Most musicians know that a minor 2nd is one key or fret and a major 2nd is two keys or frets. The keyspans of larger intervals aren't as well known. The concept can easily be expanded to other edos, but we'll assume 12-edo for now. The **stepspan** of an interval is simply the degree minus one. M2, m2, A2 and d2 all have a stepspan of 1. P5, d5 and A5 all have stepspan 4. The stepspan can be thought of as the 7-edo keyspan. This concept can be expanded to include pentatonicism, octotonicism, etc., but we'll assume heptatonicism for now.

Every 3-limit interval can be uniquely expressed as the combination of a keyspan and a stepspan. This combination is called a **gedra**, analogous to a monzo, but written in brackets not parentheses: 3/2 = (-1,1) is a 7-semitone 5th, thus (-1,1) = [7,4]. 9/8 = (-3,2) = [2,1] = a 2-semitone 1-step interval. The octave 2/1 = [12,7]. For any 3-limit interval with a monzo (a,b), there is a unique gedra [k,s], and vice versa:

k = 12a + 19b
s = 7a + 11b

The matrix [(12,19) (7,11)] is unimodular, and can be inverted, and (a,b) can be derived from [k,s]:

a = -11k + 19s
b = 7k - 12s

Gedras can be manipulated exactly like monzos. Just as adding two intervals (a,b) and (a',b') gives us (a+a',b+b'), likewise [k,s] added to [k',s'] equals [k+k',s+s']. If the GCD of a and b is n, then (a,b) is a stack of n identical intervals, with (a,b) = (na', nb') = n(a',b'), and if (a,b) is converted to [k,s], then the GCD of k and s is also n, and [k,s] = [nk',ns'] = n[k',s'].

Gedras greatly facilitate finding a pergen's period, generator and enharmonic(s). A given fraction of a given 3-limit interval can be approximated by simply dividing the keyspan and stepspan directly, and rounding off. This approximation will usually produce an enharmonic interval with the smallest possible keyspan and stepspan, which is the best enharmonic for notational purposes. As noted above, the smaller of two equivalent periods or generators is preferred, so fractions of the form N/2 should be rounded down, not up.

For example, consider the half-5th pergen. P5 = [7,4], and half a 5th is approximately [round(7/2), round (4/2)] = [3,2] = m3. The enharmonic can also be found using gedras: xE = M - n·G = P5 - 2·m3 = [7,4] - 2·[3,2] = [7,4] - [6,4] = [1,0] = A1.

Next, consider (P8/5, P5). P = [12,7]/5 = [2,1] = M2. xE = P8 - mP = P8 - 5·M2 = [12,7] - 5·[2,1] = [2,2] = 2·[1,1] = 2·m2. Because x = 2, E will occur twice in the perchain, even thought the comma only occurs once (i.e. the comma = 2·m2 = d3). The enharmonic's **count** is 2. The bare enharmonic is m2, which must be downed to vanish. The number of downs equals the splitting fraction, thus E = v^5m2. Since P8 = 5·P + 2·E, the period must be ^^M2, to make the ups and downs come out even. The number of the period's (or generator's) ups or downs always equals the count. Equipped with the period and the enharmonic, the perchain is easily found:

P1 -- ^^M2=v^3m3 -- v4 -- ^5 -- ^{3}M6=vvm7 -- P8
C -- D^^=Ebv^3 -- Fv -- G^ -- A^3=Bbvv -- C

Most single-split pergens are dealt with similarly. For example, (P8, P4/5) has a bare generator [5,3]/5 = [1,1] = m2. The bare enharmonic is P4 - 5·m2 = [5,3] - 5·[1,1] = [5,3] - [5,5] = [0,-2] = -2·[0,1] = two descending d2's. The d2 must be upped, and E = ^{5}d2. Since P4 = 5·G - 2·E, G must be ^^m2. The genchain is:

P1 -- ^^m2=v^3A1 -- vM2 -- ^m3 -- ^{3}d4=vvM3 -- P4
C -- Db^^ -- Dv -- Eb^ -- Evv -- F

To find the single-pair notation for a false double pergen, find an explicitly false form of the pergen, and find the generator and enharmonic from the fractional multigen as before. Then deduce the period from the enharmonic. If the multigen was changed by unreducing, find the original generator from the period and the alternate generator.

For example, (P8/5, P4/2) isn't explicitly false, so we must unreduce it to (P8/5, m2/10). The bare alternate generator G' is [1,1]/10 = [0,0] = P1. The bare enharmonic is m2 - 10·P1 = m2. It must be downed, thus E = v^{10}m2. Since m2 = 10·G' + E, G' is ^1. The octave plus or minus some number of enharmonics must equal 5 periods, thus (P8 + x·m2) must be divisible by 5, and ([12,7] + x[1,1]) mod 5 must be 0. The smallest (least absolute value) x that satisfies this equation is -2, and P8 = 5·P + 2·E, and P = ^{4}M2. Next, find the original half-4th generator. P = P8/5 = ~240¢, and G =

P4/2 = ~250¢. Because P < G, G' is not P - G but G - P, and G is not P - G' but P + G', which equals ^^{4}M2 + ^1 = ^^{5}M2. While the alternate multigen is more complex than the original multigen, the alternate generator is usually simpler than the original generator.

P1- - ^^{4}M2=v^{6}m3 -- vvP4 -- ^^P5 -- ^^{6}M6=v^{4}m7 -- P8

C -- D^^{4}=Ebv^6 -- Fvv -- G^^ -- A^^{6}=Bbv^4 -- C

P1 -- ^^{5}M2=v^{5}m3 -- P4

C -- D^^{5}=Ebv^5 -- F

To find the double-pair notation for a true double pergen, find each pair from each half of the pergen. Each pair has its own enharmonic. For (P8/2, P4/2), the split octave implies P = vA4 and E = ^^d2, and the split 4th implies G = /M2 and E' = \\m2.

A false-double pergen can optionally use double-pair notation, which is found as if it were a true double. Double-pair notation is often preferable when the single-pair enharmonic is not a unison or a 2nd, as with (P8/2, P4/3).

Even single-split pergens may benefit from double-pair notation. For example, (P8, P11/4) has an enharmonic that's a 3rd: P11/4 = [17,10]/4 = [4,2] = M3, and P11 - 4·M3 = [1,2] = dd3. So E = v^{4}dd3, and G = ^M3. But by using double-pair notation, we can avoid that. We find P11/2, which equals two generators: P11/2 = 2·G = [17,10]/2 = [8,5] = m6. The bare enharmonic is P11 - 2·m6 = [1,0] = A1. For this second enharmonic, we use the second pair of accidentals: E' = \\A1 and 2·G = /m6 or \M6. The sum or difference of two enharmonic intervals is also an enharmonic: E + E' = v^4\\d3 = 2·vv\m2, and E - E' = v^4//ddd3 = 2·vv/d2. Thus vv\m2 and vv/d2 are equivalent enharmonics, and v\4 and v/d4 are equivalent generators. Here is the genchain:

P1 -- ^M3=v\4 -- /m6=\M6 -- ^/8=vm9 -- P11

C -- E^=Fv\ -- Ab/=A\ -- C^/=Dbv -- F

One might think with (P8/3, P11/4), that one pair would be needed to split the octave, and another two pairs to split the 11th, making three in all. But only two pairs are needed. First unreduce to get (P8/3, m3/12). m3/12 is the alternate generator G'. We have [3,2]/12 = [0,0] = P1, and G' = ^1 and E = v^{12}m3. Next find 4·G' = m3/3 = [3,2]/3 = [1,1] = m2. Next, the bare enharmonic: m3 - 3·m2 = [0,-1] = descending d2. Thus E' = /^{3}d2, and 4·G' = /m2. The period can be deduced from 4·G': P8/3 = (m10 - m3)/3 = (m10)/3 - 4·G' = P4 - /m2 = \M3. From the unreducing, we know that G - P = P11/4 - P8/3 = m3/12, so G = P11/4 = P8/3 + m3/12 = \M3 + ^1 = ^\M3. Equivalent enharmonics are found from E + E' and E - 2·E'. Equivalent periods and generators are found from the many enharmonics, which also allow much freedom in chord spelling.
Enharmonic = v^{12}m3 = /^{3}d2 = v^4/m2 = v^4\\A1. Period = \M3 = v^{4}4 = //d4. Generator = ^\M3 = v^{3}4 = ^//d4.

P1 — \M3 — \\A5=/m6 — P8

C — E\ — Ab/ — C

P1 — ^\M3 — ^^\\A5=^^/m6=vv\M6 — ^^{3}8=v/m9 — P11

C — E^\ — Ab^^/=Avv\ — Dbv/ — F

It's not yet known if every pergen can avoid large enharmonics (those of a 3rd or more) with double-pair notation. One situation in which very large enharmonics occur is the "half-step glitch". This is when the stepspan of the multigen is half (or a third, a quarter, etc.) of the multigen's splitting fraction. For example, in sixth-4th, six generators must cover three scale steps, and each one must cover a half-step. Each generator is either a unison or a 2nd, which causes the enharmonic's stepspan to equal the multigen's stepspan.

Sixth-4th with single-pair notation has an awkward ^^{6}d^{6}4 enharmonic. It might result from combining half-4th and third-4th (e.g. tempering out both the semaphore and porcupine commas), and its double-pair notation can also combine both. Half-4th has E = vvm2 and G = ^M2 = vm3. Third-4th has E' = \$^3$A1 and G' = \M2 = //m2. G - G' = P4/2 - P4/3 = P4/6. Thus the sixth-4th generator is G - G' = ^M2 - \M2 = ^/1.

P1 — ^/1=v/m2 — //m2=\M2 — ^M2=vm3 — /m3=\\M3 — ^\M3=v\4 — P4

C — C^/=Dbv/ — Db//=D\ — D^=Ebv — Eb/=E\\ — E^/=Fv\ — F

Alternate enharmonics

Sometimes the enharmonic found by rounding off the gedra can be greatly improved by rounding off differently. For example, (P8/3, P4/4) unreduces to (P8/3, WWM6/12), a false double. The bare alternate generator is WWM6/12 = [33,19]/12 = [3,2] = m3. The bare enharmonic is [33,19] - 12·[3,2] = [-3,-5] = a quintuple-diminished 6th! This would make for a very confusing notation. However, [33,19]/12 can be rounded very inaccurately all the way up to [4,2] = M3. The enharmonic becomes [33,19] - 12·[4,2] = [-15,-5] = -5·[3,1] = -5·v^{12}A2, which is an improvement but still awkward. The period is ^^{4}m3 and the generator is v^{3}M2. ^1 = 25¢ + 0.75·c, about an eighth-tone.

P1 -- ^^{4}m3 -- v^{4}M6 -- C

C -- Eb^^{4} -- Av^4 -- C

P1 -- v^{3}M2 -- v^{6}M3=^^{6}m2 -- ^^{3}m3 -- P4

C -- Dv^3 -- Ev^6=Db^^{6} -- Eb^^{3} -- F

Because G is a M2 and E is an A2, the equivalent generator G - E is a descending A1. Ascending intervals that look descending can be confusing, one has to take into account the eighth-tone ups to see that Dv^3 -- Db^^{6} is ascending. Double-pair notation may be preferable. This makes P = vM3, E = ^3d2, G = /m2, and E' = /4dd2.

P1 -- vM3 -- ^m6 -- P8

C -- Ev -- Ab^ -- C

P1 -- /m2 -- //d3=\\A2 -- \M3 -- P4

C -- Db/ -- Ebb//=D#\\ -- E\ -- F

Because the enharmonic found by rounding off the gedra is only an estimate that may need to be revised, there isn't any point to using gedras with something other than 12-edo keyspans, like 5edo or 19edo. Heptatonic stepspans are best because conventional notation is heptatonic, and we want to minimize the heptatonic stepspan of the enharmonic.

To search for alternate enharmonics, convert E to a gedra, then multiply it by the count to get the multi-E (multi-enharmonic). The count is always the number of ups or downs in the generator (or period). The count is positive only if G (or P) is upped and E is downed, or vice versa. Add or subtract the splitting fraction n (or m) to/from either half of the gedra as desired to get a new multi-E. If the stepspan becomes negative, or if it's zero and the keyspan becomes negative, invert the gedra. If the two halves of the new gedra have a common factor, simplify the gedra by this factor, which becomes the new count. Convert the simplified gedra to a 3-limit interval. Add n (or m) ups or downs, this is the new E. Choose between ups and downs according to whether the 5th falls in the enharmonic's upping or downing range, see tipping points above. Add n·count ups or downs to the new multi-E. Add or subtract the new multi-E from the multigen (or the octave) to get an interval which splits cleanly into n (or m) parts. Each part is the new generator (or period).

For example, (P8, P5/3) has n = 3, G = ^M2, and E = v^{3}m2 = [1,1]. G is upped only once, so the count is 1, and the multi-E is also [1,1]. Subtract n from the gedra's keyspan to make a new multi-E [-2,1]. This can't be simplified, so the new E is also [-2,1] = d^{3}2. Assuming a reasonably just 5th, E needs to be upped, so E = ^^{3}d^{3}2. Add the multi-E ^^{3}[-2,1] to the multigen P5 = [7,4] to get ^^{3}[5,3]. This isn't divisible by n, so we must subtract instead: [7,4] - ^^{3}[-2,1] = v^3[9,3] = 3·v[3,1], and G = vA2. Use the equation M = n·G + y·E to check: 3·vA2 + 1·^3d^{3}2 = 3·M2 + 1·m2 = P5. (Diminish three A2's once and augment one d^{3}2 three times.) Here are the genchains: P1 -- vA2=^^dd3 -- ^d4 -- P5 and C -- D#v -- Fb^ -- G. Because d^{3}2 = -200¢ - 26·c, ^ = (-d^{3}2) / 3 = 67¢ + 8.67·c, about a third-tone.

Approaching pergens in a higher-primes-agnostic way, independently of specific commas or temperaments, one enharmonic (and one notation) will usually be obviously superior. But sometimes the temperament being notated implies a certain enharmonic. Specifically, the comma tempered out should map to E, or the multi-E if the count is > 1. For example, consider semaphore (2.3.7 and 49/48), which is half-4th. Assuming 7/4 is a m7, the comma is a m2, and a vvm2 enharmonic makes sense. G = ^M2 and the genchain is C -- D^=Ebv -- F. But consider double large deep yellow, which tempers out (-22, 11, 2). This temperament is also half-4th. The comma is a descending dd2, thus E = ^^dd2, G = vA2, and the genchain is C -- D#v=Ebb^ -- F. This is the best notation because the (-10, 5, 1) generator is an augmented 2nd.

Of course, the 2nd comma is much more obscure than 49/48, and much harder to pump. Most half-4th commas are in fact minor 2nds, and the vvm2 enharmonic is indeed a superior notation. But there is another situation in which alternate harmonics arise. There is no consensus on how to map primes 11 and 13 to the 3-limit. 11/8 can be either P4 or A4, and 13/8 can be either m6 or M6. The choice can affect the choice of enharmonic.

For example, small triple amber tempers out (12,-1,0,0,-3) from 2.3.11, making a third-11th pergen. The generator is 11/8. If 11/8 is notated as an ^4, the enharmonic is v^{3}M2, but if 11/8 is notated as a vA4, the enharmonic is ^^{3}dd2.

Sometimes the temperament implies an enharmonic that isn't even a 2nd. For example, liese (2.3.5.7 with 81/80 and 1029/1000) is (P8, P11/3), with G = 7/5 = vd5, and E = 3·vd5 - P11 = v^{3}dd3. The genchain is P1 -- vd5 -- ^M7 -- P11, or C -- Gbv -- B^ -- F.

This is all for single-comma temperaments. Each comma of a multiple-comma temperament also implies an enharmonic, and they may conflict. True double pergens, which are always multi-comma, have multiple notations. For example, the half-everything pergen has three possible notations, all equally valid. Even single-split pergens can have multiple commas that imply different enharmonics.

Chord names and staff notation

Using pergens, all rank-2 chords can be named using ups and downs, and if needed highs and lows as well. See the ups and downs page for chord naming conventions. The genchain and/or the perchain creates a lattice in which each note and each interval has its own name. The many enharmonic equivalents allow proper chord spelling.

In certain pergens, one spelling isn't always clearly better. For example, in half-4th, C E G A^ and C E G Bbv are the same chord, and either spelling might be used. This exact same issue occurs in 24-edo.

Given a specific temperament, the full period/generator mapping gives the notation of higher primes, and thus of any ratio. Thus JI chords can be named. For example, pajara (2.3.5.7 with 50/49 and 64/63) is (P8/2, P5), half-8ve, with P = vA4 or ^d5, G = P5, and E = ^^d2. The full mapping is [(2 2 7 8) (0 1 -2 -2)], which can also be written [(2 0) (2 1) (7 -2) (8 -2)]. This tells us 7/1 = 8·P - 2·G = 4·P8 - 2·P5 = WWm7, and 7/4 = m7. Likewise 5/1 = 7·P - 2·G = 7/1 minus a half-octave. From this it follows that 5/4 = m7 - ^d5 = vM3. A 4:5:6:7 chord is written C Ev G Bb = C7(v3).

A different temperament may result in the same pergen with the same enharmonic, but may still produce a different name for the same chord. For example, injera (2.3.5.7 with 81/80 and 50/49) is also half-8ve. However, the tipping point for the d2 enharmonic is at 700¢, and while pajara favors a fifth wider than that, injera favors a fifth narrower than that. Hence ups and downs are exchanged, and E = vvd2, and P = ^A4 = vd5. The mapping is [(2 2 0 1) (0 1 4 4)] = [(2 0) (2 1) (0 4) (1 4)]. Because the square mapping (the first two columns) are the same, the pergen is the same. Because the other columns are different, the higher primes are mapped differently. 5/4 = M3 and 7/4 = M3 + vd5 = vm7, and 4:5:6:7 = C E G Bbv = C,v7.

Scales can be named similar to Meantone[7], as (P8, P5) [7] = unsplit heptatonic, or (P8, P5/2) [5] = half-fifth pentatonic, etc. The number of notes in the scale tend to be a multiple of m, e.g. half-octave pergens tend to have scales with an even number of notes.

Chord progressions can be written out by applying ups and downs to the chord roots as needed, e.g. I.v -- vIII.v -- vVI.^m -- I.v. A porcupine (third-4th) comma pump can be written out like so: C.v -- Av.^m -- Dv.v -- [Bvv=Bb^]^m -- Eb^.v -- G.^m -- G.v -- C.v. Brackets are used to show that Bvv and Bb^ are enharmonically equivalent. The equivalence is shown roughly half-way through the pump. Bvv is written first to show that this root is a vM6 above the previous root, Dv. Bb^ is second to show the P4 relationship to the next root, Eb^. Such an equivalence of course couldn't be used on the staff, where the chord would be written as either Bvv.^m or Bb^.^m, or possibly Bb^.vvM = Bb^ Dv F^.

Highs and lows, like ups and downs, precede the note head and any sharps or flats. Scores for melody instruments can optionally have them above or below the staff. This score uses ups and downs, and has chord names.

Mizarian Porcupine Overture by Herman Miller (P8, P4/3)

Tipping points and sweet spots

The tipping point for half-octave with a d2 enharmonic is 700¢, 12-edo's 5th. As noted above, the 5th of pajara (half-8ve) tends to be sharp, thus it has E = ^^d2. But injera, also half-8ve, has a flat 5th, and thus E = vvd2. It is fine for two temperaments with the same pergen to be on opposite sides of the tipping point. But if a single temperament "tips over", either the up symbol sometimes means down in pitch, or even worse, the direction of ups and downs for a piece would reverse if the tuning is adjusted slightly. Fortunately, the temperament's "sweet spot", where the damage to those JI ratios likely to occur in chords is minimized, rarely contains the tipping point.

The tipping point depends on the choice of enharmonic. It's not the temperament that tips, it's the notation. Half-8ve could be notated with an E of vvM2. The tipping point becomes 600¢, a very unlikely 5th, and tipping is impossible. For single-comma temperaments, E usually equals the 3-limit mapping of the comma. Thus for 5-limit and 7-imit temperaments, the choice of E is a given. However, the mapping of primes 11 and 13 is not agreed on.

The notation's tipping point is determined by the bare enharmonic, which is implied by the vanishing comma. For example, porcupine's 250/243 comma is an A1 = (-11,7), which implies a bare E of A1, which implies 7-edo, and a 685.7¢ tipping point. Dicot's 25/24 comma is also an A1, and has the same tipping point. Semaphore's 49/48 comma is a minor 2nd = (8,-5), implying 5-edo and a 720¢ tipping point. See tipping points above for a more complete list.

Double-pair notation has two enharmonics, and two tipping points to be avoided. (P8/2, P4/2) has three possible notations. The two enharmonics can be either A1, m2 or d2. One can choose to use whichever two enharmonics best avoid tipping.

An example of a temperament that tips easily is negri, 2.3.5 and (-14,3,4). Because negri is 5-limit, the mapping is unambiguous, and the comma must be a dd2, implying 19-edo and a 694.74¢ tipping point. This coincides almost exactly with negri's seventh-comma sweet spot, where 6/5 is just and 3/2 = 694.65¢. Negri's pergen is quarter-4th, with ^ = 25¢ + 4.75c, very nearly 0¢. The up symbol represents either 81/80 or 80/81, and E could be either ^^{4}dd2 or v^{4}dd2. When the choice is so arbitrary, it's perhaps best to avoid inverting the ratio. 81/80 implies an E of ^^{4}dd2 and a G of ^m2. Negri's generator is 16/15, which is indeed a m2 raised by 81/80. In practice, seventh-comma negri's 5th is only 0.085¢ from 19-edo's 5th, the tuning is hardly audibly different than 19edo, and the piece can be written out in 19edo notation.

Another "tippy" temperament is found by adding the mapping comma 81/80 to the negri comma and getting the large triple yellow comma (-18,7,3). This temperament is (P8, P11/3) with G = 45/32. The sweet spot is tenth-comma, even closer to 19edo's 5th.

Notating unsplit pergens

An unsplit pergen doesn't require ups and downs, but they are generally needed for proper chord spellings. The only exception is when tempering out a mapping comma, such as 81/80 or 64/63. For single-comma rank-2 temperaments, the pergen is unsplit if and only if the vanishing comma's color depth is 1 (i.e. the monzo has a final exponent of ±1).

The following table shows how to notate various 5-limit rank-2 temperaments. The sweet spot isn't precisely defined, thus all cents are approximate. The up symbol's ratio is always the mapping comma, or its inverse.

5-limit temperament	comma	sweet spot	no ups or downs		with ups and downs		up symbol		
(pergen is unsplit)		(5th = 700¢ + c)	5/4 is	4:5:6 chord	5/4 is	4:5:6 chord	E	ratio	cents
meantone	81/80 = P1	c = -3¢ to -5¢	M3	C E G	---	---	---	---	---
mavila	135/128 = A1	c = -21¢ to -22¢	m3	C Eb G	^M3	C E^ G	^A1	80/81 = d1	-100¢ - 7c = 47¢-54¢
large green	(-15,11,-1) = A1	c = -10¢ to -12¢	A3	C E# G	^M3	C E^ G	vA1	80/81 = A1	100¢ + 7c = 26¢-30¢
schismic	(-15,8,1) = -d2	c = 1.7¢ to 2.0¢	d4	C Fb G	vM3	C Ev G	^d2	81/80 = -d2	12c = 20¢-24¢
double large green	(-23,16,-1) = -d2	c = -0.9¢ to -1.2¢	AA2	C D## G	vM3	C Ev G	vd2	81/80 = d2	-12c = 10¢-15¢
father	16/15 = m2	c = 56¢ to 58¢	P4	C F G	vM3	C Ev G	^m2	81/80 = -m2	-100¢ + 5c = 180-190¢
superpyth	(12,-9,1) = m2	c = 9¢ to 10¢	A2	C D# G	vM3	C Ev G	vm2	81/80 = m2	100¢ - 5c = 50-55¢

The schismic comma is a negative (i.e. descending) dim 2nd because it takes you down the scale, but up in pitch. The mavila temperament could perhaps be notated without ups and downs, because 5/4 is still a 3rd, and the 4:5:6 triad still looks like a triad.

For unsplit pergens only, the up symbol's ratio can be expressed as a 3-limit comma, which is the sum or difference of the vanishing comma and the mapping comma. This 3-limit comma is tempered, as is every ratio. It can also be found directly from the 3-limit mapping of the vanishing comma. For example, the schismic comma is a descending d2, and d2 = (19,-12), therefore the 3-comma is the pythagorean comma (-19,12).

A similar table could be made for 7-limit commas of the form (a,b,0,±1). Every such temperament except for archy (2.3.7 and 64/63) might use ups and downs to spell 7/4 as a m7. A 7-limit temperament with two commas may need double-pair notation, even though its pergen is unsplit, to avoid spelling the 4:5:6:7 chord something like C D# G A#. However, if both commas map to the same 3-limit comma, only single-pair is needed. For example, 7-limit schismic tempers out both (-15,8,1) = -d2 and (25,-14,0,-1) = d2. The up symbol stands for the pythagorean comma, and the 4:5:6:7 chord is spelled C Ev G Bbv.

Notating rank-3 pergens

Conventional notation is generated by the octave and the 5th, and the notation (not the tuning itself) is rank-2. Each additional pair of accidentals increases the notation's rank by one. Enharmonics are like commas in that each one reduces the notation's rank by one. Obviously, the notation's rank must match the actual tuning's rank. Therefore the minimum number of enharmonics needed always equals the difference between the notation's rank and the tuning's rank. Examples:

tuning	pergen	tuning's rank	notation	notation's rank	# of enharmonics needed	enharmonics
12-edo	(P8/12)	rank-1	conventional	rank-2	1	E = d2
19-edo	(P8/19)	rank-1	conventional	rank-2	1	E = dd2
15-edo	(P8/15)	rank-1	single-pair	rank-3	2	E = m2, E' = v3A1 = v3M2
24-edo	(P8/24)	rank-1	single-pair	rank-3	2	E = d2, E' = vvA1 = vvm2
pythagorean	(P8, P5)	rank-2	conventional	rank-2	0	---
meantone	(P8, P5)	rank-2	conventional	rank-2	0	---
srutal	(P8/2, P5)	rank-2	single-pair	rank-3	1	E = ^^d2
semaphore	(P8, P4/2)	rank-2	single-pair	rank-3	1	E = vvm2
decimal	(P8/2, P4/2)	rank-2	double-pair	rank-4	2	E = vvd2, E' = \\m2 = ^^\\A1
5-limit JI	(P8, P5, ^1)	rank-3	single-pair	rank-3	0	---
marvel	(P8, P5, ^1)	rank-3	single-pair	rank-3	0	---
breedsmic	(P8, P5/2, ^1)	rank-3	double-pair	rank-4	1	E = \\dd3
7-limit JI	(P8, P5, ^1, /1)	rank-4	double-pair	rank-4	0	---

When there is more than one enharmonic, the first one can be added to or subtracted from the 2nd one, to make an equivalent 2nd enharmonic.

A rank-2 pergen is either unsplit, single-split or double-split, and a double-split is either a true double or a false double. A rank-3 pergen can be any of these. Additionally, some rank-3 pergens are triple-splits, which are either true triples or false triples. False triples are like true doubles in that they only require two commas. There are even "superfalse" triples that can arise from a single comma, but the higher prime's exponent in the comma must be at least 12, making it difficult to pump, and not very useful musically.

Even the unsplit rank-3 pergen requires single-pair notation, for the 2nd generator. Single-splits and false double-splits require double-pair, true doubles and false triples require triple-pair, and true triples require quadruple-pair. Some false triples and some true doubles may use quadruple-pair as well, to avoid awkward enharmonics of a 3rd or more. Rather than devising a third or fourth pair of symbols, and a third or fourth pair of adjectives to describe them, one might simply use colors.

A true/false test hasn't yet been found for either triple-splits, or double-splits in which multigen2 is split.

Some examples of 7-limit rank-3 temperaments:

7-limit temperament	comma	pergen	spoken pergen	notation	period	gen1	gen2	enharmonic
marvel	225/224	(P8, P5, ^1)	rank-3 unsplit	single-pair	P8	P5	^1 = 81/80	---
"	"	"	"	double-pair	"	"	"	^^\d2
deep reddish	50/49	(P8/2, P5, ^1)	rank-3 half-8ve	double-pair	v/A4 = 10/7	P5	^1 = 81/80	^^\\d2
triple bluish	1029/1000	(P8, P11/3, ^1)	rank-3 third-11th	double-pair	P8	^\d5 = 7/5	^1 = 81/80	^^^\\\dd3
breedsmic	2401/2400	(P8, P5/2, ^1)	rank-3 half-5th	double-pair	P8	v//A2 = 60/49	/1 = 64/63	^^\4dd3

demeter	686/675	(P8, P5, vm3/2)	half-downminor-3rd	double-pair	P8	P5	v/A1 = 15/14	^^\\\dd3

If using single-pair notation, marvel is notated like 5-limit JI, but the 4:5:6:7 chord is spelled C - Ev - G - A#vv. If double-pair notation is used, /1 = 64/63, and the chord is spelled C - Ev - G - Bb\.

There's a lot of options in rank-3 double-pair notation for what ratio each accidental pair represents. For example, deep reddish is half-8ve, with a d2 comma, like srutal. Using the same notation as srutal, but with ^1 = 81/80, we have P = \A4 = /d5 and E = //d2. The ratio for /1 is (-10,6,-1,1), a descending interval. 7/4 = 5/4 + 7/5 = vM3 + /d5 = v/m7, and the 4:5:6:7 chord is spelled awkwardly as C Ev G Bbv/. However, double-pair notation for 7-limit rank-3 temperaments can be standardized so that ^1 is always 81/80 and /1 is always 64/63. This ensures the 4:5:6:7 chord is always spelled C Ev G Bb\.

With this standardization, the enharmonic can be derived directly from the comma. The vanishing comma is written as some 3-limit comma plus some number of mapping commas. For example, 50/49 = (81/80)$^{-2}$ · (64/63)^{2} · (-19,12). This can be rewritten as vv1 + //1 - d2 = vv//-d2 = -^^\\d2. The comma is negative (i.e.descending), but the enharmonic never is, therefore E = ^^\\d2. The period is found by adding/subtracting E from the 8ve and dividing by two, thus P = v/A4 = ^\d5. Both E and P become more complex, but the ratio for /1 becomes simpler.

This 3-limit comma defines the tipping point. At the tipping point, the 3-limit comma vanishes too. In a rank-2 temperament, the mapping comma must also vanish, because some number of them plus the 3-limit comma must add up to the original comma, which vanishes. However, a rank-3 temperament has two mapping commas, and neither is forced to vanish if the 3-limit comma vanishes. A rank-3 double-pair notation's tipping point is where both mapping commas are tempered out. For deep reddish, this happens when the tuning is exactly 12edo. This tuning is much farther from just than need be, well outside the sweet spot. Therefore deep reddish doesn't tip. Single-pair rank-3 notation has no enharmonic, and thus no tipping point. Double-pair rank-3 notation has 1 enharmonic, but two mapping commas. Rank-3 notations rarely tip.

Unlike the previous examples, Demeter's gen2 can't be expressed as a mapping comma. It divides 5/4 into three 15/14 generators, and 7/6 into two generators. Its pergen is (P8, P5, vm3/2). It could also be called (P8, P5, vM3/3), but the pergen with a smaller fraction is preferred. Because the 8ve and 5th are unsplit, single-pair notation is possible, with gen2 = ^m2 and no E. But the 4:5:6:7 chord would be spelled C -- Fbbb^^^ -- G -- Bbb^^, very awkward! Standard double-pair notation is better. Gen2 = v/A1, E = ^^\\\dd3, and C^^\\\ = A##. Genchain2 is C -- C#v/ -- Eb\ -- Ev -- Gb\\ -- Gv\ -- G#vv=Bbb\\\ -- Bbv\\... Unlike other genchains we've seen, the additional accidentals get progressively more complex. Whenever an accidental has its own enharmonic, with no other accidentals in it, it always adds up to something simpler eventually. If it doesn't have its own enharmonic, it's infinitely stackable. A case can be made for a convention that colors are used only for infinitely stackable accidentals, and ups/downs/highs/lows only for the other kind of accidentals.

There are always many alternate 2nd generators. Any combination of periods, 1st generators and commas can be added to or subtracted from gen2 to make alternates. If gen2 can be expressed as a mapping comma, that is preferred. For demeter, any combination of vm3, double-8ves and double-5ths (M9's) makes an alternate multigen2. Any 3-limit interval can be added or subtracted twice, because the splitting fraction is 2. Obviously we can't choose the multigen2 with the smallest cents, because there will always be a 3-limit comma small enough to be subtracted twice from it. Instead, once the splitting fraction is minimized, choose the multigen2 with the smallest odd limit. In case of two ratios with the same odd limit, as 5/3 and 5/4, the **DOL** (double odd limit) is minimized. DOL (5/3) = (5,3) and DOL (5/4) = (5,1). Since 1 < 3, 5/4 is preferred.

If ^1 = 81/80, possible half-split gen2's are vM3/2, vM6/2, and their octave inverses ^m6/2 and ^m3/2. Possible third-split gen2's are vM3/3, vM6/3, vM2/3, and their inverses, plus vM9/3, ^m10/3 and vM10/3. If ^1 = 64/63, possible third-splits are ^M2/3, vm3/3, ^M3/3, vm6/3, ^M6/3, vm7/3, ^M9/3, vm10/3 and ^M10/3. Analogous to rank-2 pergens with imperfect multigens, there will be occasional double-up or double-down multigen2's.

All possible rank-3 pergens can be listed, but the table is much longer than for rank-2 pergens. Here are all the half-split pergens:

pergen number	prime subgroup			
unsplit	**2.3.5 (^1 = 81/80)**		**2.3.7 (^1 = 64/63)**	
1	(P8, P5, ^1)	rank-3 unsplit	same	same
half-splits				
2	(P8/2, P5, ^1)	rank-3 half-8ve	same	same
3	(P8, P4/2, ^1)	rank-3 half-4th	same	same
4	(P8, P5/2, ^1)	rank-3 half-5th	same	same
5	(P8/2, P4/2, ^1)	rank-3 half-everything	same	same
6	(P8, P5, ^m3/2)	half-upminor-3rd	(P8, P5, ^M2/2)	half-upmajor-2nd
7	(P8, P5, vM3/2)	half-downmajor-3rd	(P8, P5, vm3/2)	half-downminor-3rd
8	(P8, P5, ^m6/2)	half-upminor-6th	(P8, P5, ^M6/2)	half-upmajor-6th
9	(P8, P5, vM6/2)	half-downmajor-6th	(P8, P5, vm7/2)	half-downminor-7th
10	(P8/2, P5, ^m3/2)	half-8ve half-upminor 3rd	(P8/2, P5, ^M2/2)	half-8ve half-upmajor-2nd
11	(P8/2, P5, vM3/2)	half-8ve half-downmajor 3rd	(P8/2, P5, vm3/2)	etc.
12	(P8/2, P5, ^m6/2)	half-8ve half-upminor 6th	(P8/2, P5, ^M6/2)	
13	(P8/2, P5, vM6/2)	half-8ve half-downmajor 6th	(P8/2, P5, vm7/2)	
14	(P8, P4/2, ^m3/2)	half-4th half-upminor 3rd	(P8, P4/2, ^M2/2)	
15	(P8, P4/2, vM3/2)	etc.	(P8, P4/2, vm3/2)	
16	(P8, P4/2, ^m6/2)		(P8, P4/2, ^M6/2)	
17	(P8, P4/2, vM6/2)		(P8, P4/2, vm7/2)	
18	(P8, P5/2, ^m3/2)		(P8, P5/2, ^M2/2)	
19	(P8, P5/2, vM3/2)		(P8, P5/2, vm3/2)	
20	(P8, P5/2, ^m6/2)		(P8, P5/2, ^M6/2)	
21	(P8, P5/2, vM6/2)		(P8, P5/2, vm7/2)	
22	(P8/2, P4/2, vM3/2)	half-everything half-downmajor-3rd	(P8/2, P4/2, ^M2/2)	

There are at least 100 third-splits and 287 quarter-splits. More columns could be added for ^1 = 33/32, ^1 = 729/704, ^1 = 27/26, etc.

Notating Blackwood-like pergens

A Blackwood-like temperament is rank-2 and equates some number of 5ths to some number of 8ves, thus equating the 5th to some exact fraction of the octave. The 5th is not independent of the octave, thus it doesn't appear in the pergen. Such pergens make a lot of sense musically when the octave's splitting fraction corresponds to an edo with a 5th fairly close to just, like P8/5, P8/7, P8/10 and especially P8/12. Pergens which imply an edo which doesn't have a decent 5th, e.g. P8/3, P8/4, P8/6, etc., are covered in the next section.

A Blackwood-like pergen is a rank-3 pergen plus a 3-limit comma. Adding this comma splits the octave and removes the middle term from the pergen. For example, Blackwood is 5-limit JI = (P8, P5, ^1) plus 256/243, making (P8/5, ^1). Such a pergen is in effect multiple copies of an edo. Its spoken name is rank-2 N-edo, meaning an edo extended to rank-2. Its notation is based on the edo's notation, expanded with an additional microtonal accidental pair. Examples:

temperament	pergen	spoken name	enharmonics	perchain	genchain	^1 ratio	/1 ratio
Blackwood	(P8/5, ^1)	rank-2 5-edo	E = m2	D E=F G A B=C D	D F#v=Gv Bvv...	81/80 = 16/15	---
Whitewood	(P8/7, ^1)	rank-2 7-edo	E = A1	D E F G A B C D	D F^ A^^...	80/81 = 135/128	---
10edo+yellow	(P8/10, /1)	rank-2 10-edo	E = m2, E' = vvA1 = vvM2	D D^=Ev E=F F^=Gv G...	D F#\=G\ B\\...	(see below)	81/80
12edo+jade	(P8/12, ^1)	rank-2 12-edo	E = d2	D D#=Eb E F F#=Gb...	D G^ C^^	33/32	---
"	"	"	"	"	D G#v=Abv Dvv...	729/704	---
17edo+yellow	(P8/17, /1)	rank-2 17-edo	E = dd3, E' = vm2 = vvA1	D D^=Eb D#=Ev E F...	D F#\ A#\\=Bv\\...	256/243	81/80

If the edo's notation uses ups and downs, the up symbol can often be equated to a 3-limit ratio. In 17-edo and 22-edo, ^1 = m2. In 31-edo and 43-edo it's d2. But in edos like 10, 15, 21 and 24, in which the circle of 5ths skips some notes, there is no 3-limit ratio. The ratio depends on the JI interpretation of the edo. For 10-edo, ^1 might equal 16/15, or 12/11, or 13/12.

The additional accidental has an equivalent ratio, found by adding the pergen's 3-limit comma onto the ratio. Blackwood's comma is 256/243, and Blackwood's ^1 is 81/80 or equivalently, 16/15.

All Blackwood-like pergens are of the form (P8/m, ^1) or (P8/m, /1), and they can be identified solely by their splitting fraction. Blackwood-like pergens are a small minority of rank-2 pergens.

It's possible to have a fifth-8ve pergen with an independent 5th, but there will be small intervals of about 20¢. Here are two such:

temperament	subgroup	comma	pergen	spoken name	enharmonic	perchain	genchain	^1 ratio
large quintuple blue	2.3.7	(-14,0,0,5)	(P8/5, P5)	fifth-8ve	E = v5m2	D E^^ Gv A^ Cvv D	C G D A E...	49/48
small quintuple red	2.3.7	(22,-5,0,-5)	"	"	"	"	"	64/63

Unlike Blackwood, the ups and downs are in the perchain, not the genchain. It would be possible to notate Blackwood similarly. The pergen would be not (P8/5, ^1), but (P8/5, M3). The perchain would be C D^^ Fv G^ Bbvv C and the genchain would be C E G#... But this is not recommended, because it would cause "missing notes" (see next section). A Blackwood-like pergen should never have an uninflected genchain.

Notating non-8ve and no-5ths pergens

In Blackwood-like pergens, the 5th is present but not independent. In no-5ths pergens, the 5th is not present, and the prime subgroup doesn't contain 3.

In any notation, every note has a name, and no two notes have the same name. A note's representation on the musical staff follows from its name. If the notation has any enharmonics, each note has several names. Generally, every name has a note. Every possible name, and anything that can be written on on the staff, corresponds to one and only one note in the lattice formed by perchains and genchains.

But in non-8ve and no-5ths pergens, not every name has a note. For example, deep reddish minus white (2.5.7 and 50/49) is (P8/2, M3) = half-8ve, major 3rd. The genchain runs C - E - G# - B# - D##... and the perchain runs C - F#v - C. There is no G or D or A note, in fact 75% of all possible note names have no actual note. 75% of all intervals don't exist. There is no perfect 5th or major 2nd. There are missing notes and missing intervals.

Conventional notation assumes the 2.3 prime subgroup. Non-8ve and no-5ths pergens can be notated in a backwards compatible way as a subset of a larger prime subgroup which contains 2 and 3. Thus 5/4 = M3, 7/4 = m7, etc. The advantage of this approach is that conventional staff notation can be used. A violinist or vocalist will immediately have a rough idea of the pitch. The disadvantage is that there is a huge number of missing notes and intervals. The composer may want to use a notation that isn't backwards compatible for composing, but use one that is for communicating with other musicians.

Just as all rank-2 pergens in which 2 and 3 are present and independent can be numbered, so can all 2.5 pergens, all 2.7 pergens, all 3.5 pergens, etc. Every rank-2 pergen can be identified by its prime subgroup and its pergen number. The pergens are grouped into blocks and sections as before. Within each section, the pergens are ordered by cents size of the generator. Sometimes the pergen can be simplified. For example, 2.7 pergen #4 is (P8, m7/2) which is (P8, P4), which is equivalent to (P8, P5). P5 represents not 3/2 but half of 16/7, which is 3/2 sharpened by half of 64/63. Simplified pergens are marked with an asterisk.

pergen number	prime subgroup					
unsplit	2.3	2.5 (M3 = 5/4)	2.7 (M2 = 8/7)	3.5 (M6 = 5/3)	3.7 (M3 = 9/7)	5.7 (WWM3 = 5/1, d5 = 7/5)
1	(P8, P5)	(P8, M3)	(P8, M2)	(P12, M6)	(P12, M3)	(WWM3, d5)
half-splits						
2	(P8/2, P5)	(P8/2, M3)	(P8/2, M2)	(P12/2, M6)	(P12/2, M3)	(M9, d5)*
3	(P8, P4/2)	(P8, M2)*	(P8, M2/2)	(P12, M6/2)	(P12, M2)*	(WWM3, m3)*
4	(P8, P5/2)	(P8, m6/2)	(P8, P5)*	(P12, P4)*	(P12, m10/2)	(WWM3, M7)*
5	(P8/2, P4/2)	(P8/2, M2)*	(P8/2, M2/2)	(P12/2, M6/2)	(P12/2, M3/2)	(M9, m3)*
third-splits						
6	(P8/3, P5)	(P8/3, M3)	(P8/3, M2)	(P12/3, M6)	(P12/3, M3)	(WWM3/3, d5)
7	(P8, P4/3)	(P8, M3/3)	(P8, M2/3)	(P12, M6/3)	(P12, M3/3)	(WWM3, d5/3)
8	(P8, P5/3)	(P8, m6/3)	(P8, m7/3)	(P12, m7/3)	(P12, P4)*	(WWM3, WA6/3)
9	(P8, P11/3)	(P8, M10/3)	(P8, M9/3)	(P12, WWM3/3)	(P12, WM7/3)	(WWM3, WWm7/3)
10	(P8/3, P4/2)	(P8/3, M2)*	(P8/3, M2/2)	(P12/3, M6/2)	(P12/3, M2)*	(WWM3/3, m3)*
11	(P8/3, P5/2)	(P8/3. m6/2)	(P8/3, P5)*	(P12/3, P4)*	(P12/3, m10/2)	(WWM3/3, M7)*
12	(P8/2, P4/3)	(P8/2, M3/3)	(P8/2, M2/3)	(P12/2, M6/3)	(P12/2, M3/3)	(M9, d5/3)*
13	(P8/2, P5/3)	(P8/2, m6/3)	(P8/2, m7/3)	(P12/2, m7/3)	(P12/2, P4)*	(M9, WA6/3)*
14	(P8/2, P11/3)	(P8/2, M10/3)	(P8/2, M9/3)	(P12/2, WWM3/3)	(P12/2, WM7/3)	(M9, WWm7/3)*
15	(P8/3, P4/3)	(P8/3, M3/3)	(P8/3, M2/3)	(P12/3, M6/3)	(P12/3, P4)*	(WWM3/3, d5/3)

For prime subgroup p.q, the unsplit pergen has period p/1. The generator is found by dividing q by p until it's less than p/1, and period-inverting if it's more than half of p/1.

Blackwood-like pergens also appear in this table. For example, in row #33, (P8/5, ^1) replaces both 2.5 (P8/5, M3) and 2.7 (P8/5, M2). This has the advantage of avoiding missing notes. Since one fifth of 5/1 is only 25¢ from 7/5, the 5.7 (WWM3/5, d5) can optionally be replaced too.

pergen number	2.3	2.5	2.7	3.5	3.7	5.7
33	(P8/5, P5)	(P8/5, ^1)	(P8/5, ^1)	(P12/5, M6)	(P12/5, M3)	(WWM3/5, ^1)

Non-8ve pergens require octave numbers (unless on the staff of course). For example, consider the 1st 3.5 pergen, (P12, M6). The note one M6 above C1 is A1, and the note three P12's above C1 is A5. (There is no A2, A3 or A4.) Referring to a note as simply A is ambiguous.

Every rank-3 pergen can also be identified by its prime subgroup and its pergen number. A similar table could be made for all rank-3 pergens. The 2.3.5 and 2.3.7 subgroups are listed in the section on rank-3 pergens. The 2.5.7 subgroup's unsplit pergen is (P8, M3, ^M2), with ^M2 = 8/7 and ^1 = √256/245 = √(81/80 * 64/63 * 64/63) = about 38¢. The 3.5.7 subgroup's unsplit pergen is (P12, M6, ^M3), with ^M3 = 9/7 and ^1 = √6561/6125 = $\sqrt{[(81/80)^3 * (64/63)^2]}$ = about 60¢.

Pergen squares

Pergen squares, which were discovered by Praveen Venkataramana, are a way to visualize pergens squares in a way that isn't specific to any primes at all. To understand them, let's assume the standard 2.3 prime subgroup for now. The genchain runs left to right along the top and bottom sides of the square. One horizontal side of the square equals one 5th. The perchain runs up the sides of the square. One vertical side of the square equals one octave. The pergen square is the building block of the rank-2 lattice. The complete lattice is formed by tiling many squares in all directions.

For (P8, P5), the pergen square has 4 notes, shown here with octave numbers (ignore the periods).
C2 -- G2
| . . . . |
C1 -- G1

Splitting the 8ve or the multigen adds notes to the square. For (P8/2, P5), there are 6 notes. The new notes fall halfway between each 8ve:
C2 --- G2
F#v1 C#v2
C1 --- G1

A pergen square shows all alternate gens and multigens. The unreduced form of (P8/2, P5) is (P8/2, M2/2). The M2 becomes visible only after tiling the pergen square. From C2 to D2 is a M2, and C#v2 bisects it. G#v2 bisects the G2-A2 M2. Every "downed" note bisects an 8ve, a M2, a Wm7 (e.g. D1 to C3), a WM9 (e.g. C1 to D3), and many other intervals.
C2 --- G2 --- D3 --- A3
F#v1 C#v2 G#v2 D#v3
C1 --- G1 --- D2 --- A2

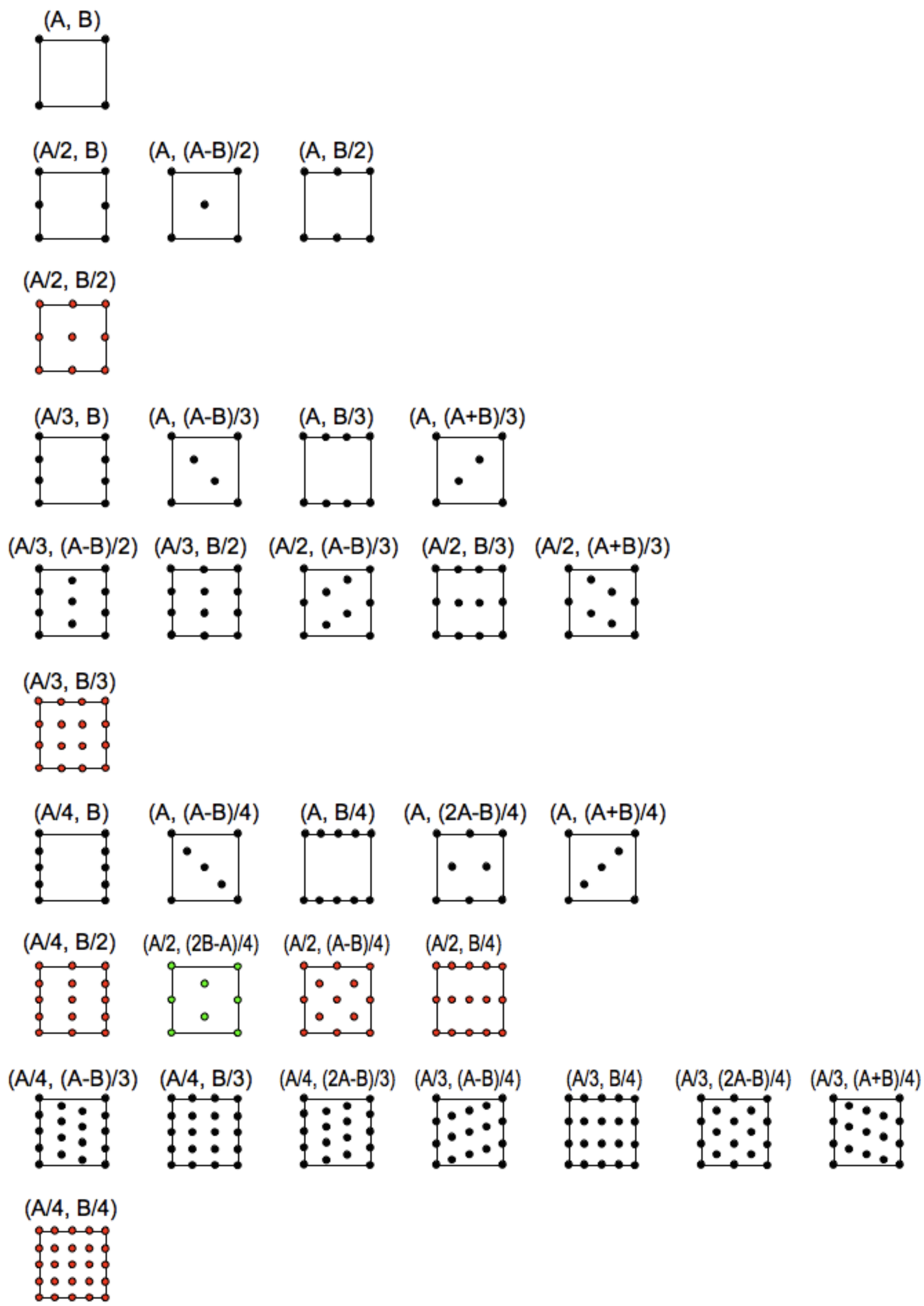
Splitting the 5th adds notes to the horizontal edges of the square:
C2 Ev2 G2
| . . . . . . |
C1 Ev1 G1

Each downed note also bisects a P11 (e.g. G1-C3), as well as many other intervals.
C3 Ev3 G3
| . . . . . . |
C2 Ev2 G2

| . . . . . . |
C1 Ev1 G1

From G1 to C2 is a 4th, so splitting the 4th adds a note halfway between them, in the center of the square.
C2 ---- G2
| . A^1 . |
C1 ---- G1
A^1 also bisects the P12 from C1 to G2.

Pergen squares can be generalized to any prime subgroup by representing the notes as dots. Below are the first 32 rank-2 pergens in a completely JI-agnostic format. A is the interval of equivalence, the period of the unsplit pergen. B is the generator of the unsplit pergen. For 2.3 pergens, A = 8ve and B = 5th. The (A, (A-B)/2) square corresponds to (P8, P4/2). In the 2.5 subgroup, B = 5/4. In Bohlen-Peirce, A = 3/1 and B = 5/3. True doubles are in red. The true/false property of a pergen is independent of the prime subgroup. Imperfect multigens are in green. Imperfect is generalized to other subgroups as requiring multiples of B in the pergen.



A similar chart could be made for all rank-3 pergens, using pergen cubes.

Notating tunings with an arbitrary generator

Given only the generator's cents, and the period as some fraction of the octave, it's often possible to work backwards and find an appropriate multigen. Heptatonic notation requires that the 5th be between 600¢ and 720¢, to avoid descending 2nds. However 600¢ makes an extremely lopsided scale, so a more reasonable lower bound of 7\13 = 647¢ is used here. This limit is chosen because 13-edo notation uses the alternate 5th 7\13, and as a bonus it includes 16/11 = 649¢. The 4th is limited to 480-553¢, which includes 11/8. This sets a range for each possible generator, e.g. half-4th's generator ranges from 240¢ to 277¢.

The next table lists all the ranges for all multigens up to seventh-splits. One can look up one's generator in the first column and find a possible multigen. Use the octave inverse if G > 600¢. Some ranges overlap. Those that are contained entirely within another range are listed in the righthand columns.

primary choice		secondary choices			
generator	possible multigen	generator	multigen	generator	multigen
23-60¢	M2/4 (requires P8/2)				
69-79¢	P4/7				
80-92¢	P4/6				
92-103¢	P5/7				
96-111¢	P4/5				
108-120¢	P5/6				
120-138¢	P4/4				
129-144¢	P5/5				
160-185¢	P4/3	162-180¢	P5/4		
215-240¢	P5/3				
240-277¢	P4/2	240-251¢	P11/7	264-274¢	P12/7
280-292¢	P11/6				
308-320¢	P12/6				
323-360¢	P5/2	336-351¢	P11/5		
369-384¢	P12/5				
411-422¢	WWP4/7				
420-438¢	P11/4				
435-446¢	WWP5/7				
462-480¢	P12/4				
480-554¢	P4 = P5	480-492¢	WWP4/6	508-520¢	WWP5/6
560-585¢	P11/3				
576-591¢	WWP4/5	583-593¢	WWWP4/7		

There are gaps in the table, especially near 150¢, 200¢, 300¢ and 400¢. Some tunings simply aren't compatible with fifth-generated heptatonic notation. But the total range of possible generators is mostly well covered, providing convenient notation options. In particular, if the tuning's generator is just over 720¢, to avoid descending 2nds, instead of calling the generator a 5th, one can call its inverse a quarter-12th. The generator is notated as an up-5th, and four of them make a WWP4.

The table assumes unsplit octaves. Splitting the octave creates alternate generators. For example, if P = P8/3 = 400¢ and G = 300¢, alternate generators are 100¢ and 500¢. Any of these can be used to find a convenient multigen.

See also the map of rank-2 temperaments.

Pergens and MOS scales

Every rank-2 pergen generates certain MOS scales. This of course depends on the exact size of the generator. In this table, the 5th is assumed to be between 4\7 and 3\5. Sometimes the genchain is too short to generate the multigen. For example, (P8/3, P4/2) [6] has 3 genchains, each with only 2 notes, and thus only 1 step. But it takes 2 steps to make a 4th, so the scale doesn't actually contain any 4ths. Such scales are marked with an asterisk.

pergen		MOS scales of 5-12 notes					
(P8, P5)	unsplit	5 = 2L 3s	7 = 5L 2s	12 = 7L 5s (or 5L 7s)			
	halves						
(P8/2, P5)	half-8ve	6 = 2L 4s	8 = 2L 6s	10 = 2L 8s	12 = 2L 10s (or 10L 2s)		
(P8, P4/2)	half-4th	5 = 4L 1s	9 = 5L 4s				
(P8, P5/2)	half-5th	7 = 3L 4s	10 = 7L 3s				
(P8/2, P4/2)	half-everything	6 = 4L 2s	10 = 4L 6s				
	thirds						
(P8/3, P5)	third-8ve	6 = 3L 3s	9 = 3L 6s	12 = 3L 9s (or 9L 3s)			
(P8, P4/3)	third-4th	5 = 1L 4s	6 = 1L 5s	7 = 1L 6s	8 = 7L 1s		
(P8, P5/3)	third-5th	5 = 1L 4s	6 = 5L 1s	11 = 5L 6s			
(P8, P11/3)	third-11th	5 = 2L 3s	7 = 2L 5s	9 = 2L 7s	11 = 2L 9s		
(P8/3, P4/2)	third-8ve, half-4th	6 = 3L 3s *	9 = 6L 3s				
(P8/3, P5/2)	third-8ve, half-5th	6 = 3L 3s *	9 = 3L 6s	12 = 3L 9s			
(P8/2, P4/3)	half-8ve, third-4th	6 = 2L 4s *	8 = 6L 2s				
(P8/2, P5/3)	half-8ve, third-5th	6 = 4L 2s *	10 = 6L 4s				
(P8/2, P11/3)	half-8ve, third-11th	6 = 2L 4s *	8 = 2L 6s	10 = 2L 8s	12 = 2L 10s		
(P8/3, P4/3)	third-everything	6 = 3L 3s *	9 = 6L 3s *				
	quarters						

(P8/4, P5)	quarter-8ve	8 = 4L 4s	12 = 4L 8s (or 8L 4s)				
(P8, P4/4)	quarter-4th	5 = 1L 4s	6 = 1L 5s	7 = 1L 6s	8 = 1L 7s	9 = 1L 8s	10 = 9L 1s
(P8, P5/4)	quarter-5th	5 = 1L 4s	6 = 1L 5s	7 = 6L 1s			
(P8, P11/4)	quarter-11th	5 = 3L 2s	8 = 3L 5s	11 = 3L 8s			
(P8, P12/4)	quarter-12th	5 = 3L 2s	8 = 5L 3s				
(P8/4, P4/2)	quarter-8ve half-4th	8 = 4L 4s *	12 = 4L 8s				
(P8/2, M2/4)	half-8ve quarter-tone	6 = 2L 4s *	8 = 2L 6s *	10 = 2L 8s	12 = 2L 10s		
(P8/2, P4/4)	half-8ve quarter-4th	6 = 2L 4s *	8 = 2L 6s *	10 = 8L 2s			
(P8/2, P5/4)	half-8ve quarter-5th	6 = 2L 4s *	8 = 6L 2s *				
(P8/4, P4/3)	quarter-8ve third-4th	8 = 4L 4s *	12 = 8L 4s *				
(P8/4, P5/3)	quarter-8ve third-5th	8 = 4L 4s *	12 = 4L 8s *				
(P8/4, P11/3)	quarter-8ve third-11th	8 = 4L 4s *	12 = 4L 8s *				
(P8/3, P4/4)	third-8ve quarter-4th	6 = 3L 3s *	9 = 3L 6s *	12 = 9L 3s *			
(P8/3, P5/4)	third-8ve quarter-5th	6 = 3L 3s *	9 = 6L 3s *				
(P8/3, P11/4)	third-8ve quarter-11th	6 = 3L 3s *	9 = 3L 6s *	12 = 3L 9s *			
(P8/3, P12/4)	third-8ve quarter-12th	6 = 3L 3s *	9 = 3L 6s *	12 = 3L 9s *			
(P8/4, P4/4)	quarter-everything	8 = 4L 4s *	12 = 8L 4s *				

A MOS scale tends to be generated by just a few pergens. The table below shows the pergen that best corresponds to each MOS scale, as well as some others that could also generate the scale. The best pergen is the simplest, and also one that makes a reasonable L/s ratio. A ratio of 3 or more makes a scale that's too lopsided.

MOS scale	**primary example**		**secondary examples**
Pentatonic			
1L 4s	(P8, P5/3) [5]	third-5th pentatonic	third-4th, quarter-4th, quarter-5th
2L 3s	(P8, P5) [5]	unsplit pentatonic	third-11th
3L 2s	(P8, P12/4) [5]	quarter-12th pentatonic	quarter-11th
4L 1s	(P8, P4/2) [5]	half-4th pentatonic	
Hexatonic			
1L 5s	(P8, P4/3) [6]	third-4th hexatonic	quarter-4th, quarter-5th, fifth-4th, fifth-5th
2L 4s	(P8/2, P5) [6]	half-8ve hexatonic	
3L 3s	(P8/3, P5) [6]	third-8ve hexatonic	
4L 2s	(P8/2, P4/2) [6]	half-everything hexatonic	
5L 1s	(P8, P5/3) [6]	third-5th hexatonic	
Heptatonic			
1L 6s	(P8, P4/3) [7]	third-4th heptatonic	quarter-4th, fifth-4th, fifth-5th, sixth-4th, sixth-5th
2L 5s	(P8, P11/3) [7]	third-11th heptatonic	fifth-double-wide-4th, sixth-double-wide-5th
3L 4s	(P8, P5/2) [7]	half-5th heptatonic	fifth-12th
4L 3s	(P8, P11/5) [7]	fifth-11th heptatonic	sixth-12th
5L 2s	(P8, P5) [7]	unsplit heptatonic	sixth-double-wide-4th
6L 1s	(P8, P5/4) [7]	quarter-5th heptatonic	
Octotonic			
1L 7s	(P8, P4/4) [8]	quarter-4th octotonic	fifth-4th, fifth-5th, sixth-4th, sixth-5th, seventh-4th, seventh-5th
2L 6s	(P8/2, P5) [8]	half-8ve octotonic	
3L 5s	(P8, P11/4) [8]	quarter-11th octotonic	seventh-WW4th, seventh-WW5th
4L 4s	(P8/4, P5) [8]	quarter-8ve octotonic	
5L 3s	(P8, P12/4) [8]	quarter-12th octotonic	(very lopsided, unless 5th is quite flat)
6L 2s	(P8/2, P4/3) [8]	half-8ve third-4th octotonic	
7L 1s	(P8, P4/3) [8]	third-4th octotonic	
Nonatonic			
1L 8s	(P8, P4/4) [9]	quarter-4th nonatonic	fifth-4th, sixth-4th, sixth-5th, seventh-4th/5th, eighth-4th/5th
2L 7s	(P8, W3P5/8) [9]	eighth-W35th nonatonic	third-11th, fifth-WW4th
3L 6s	(P8/3, P5) [9]	third-8ve nonatonic	third-8ve half-5th
4L 5s	(P8, P12/7) [9]	seventh-12th nonatonic	sixth-11th
5L 4s	(P8, P4/2) [9]	half-4th nonatonic	(lopsided unless 4th is sharp), seventh-11th
6L 3s	(P8/3, P4/2) [9]	third-8ve half-4th nonatonic	
7L 2s	(P8, WWP5/6)[9]	sixth-WW5th nonatonic	(lopsided unless 5th is sharp)
8L 1s	(P8, P5/5) [9]	fifth-5th nonatonic	
Decatonic			
1L 9s	(P8, P5/6) [10]	sixth-5th decatonic	fifth-4th, sixth-4th, seventh-4th/5th, eighth-4th/5th, ninth-4th/5th
2L 8s	(P8/2, P5) [10]	half-8ve decatonic	half-8ve quartertone, half-8ve third-11th

3L 7s	(P8, P12/5) [10]	fifth-12th decatonic	eighth-WW4th, eighth-WW5th
4L 6s	(P8/2, P4/2) [10]	half-everything decatonic	
5L 5s	(P8/2, P5) [10]	half-8ve decatonic	(lopsided unless 5th is quite flat)
6L 4s	(P8/2, P5/3) [10]	half-8ve third-5th decatonic	
7L 3s	(P8, P5/2) [10]	half-5th decatonic	ninth-WW5th
8L 2s	(P8/2, P4/4) [10]	half-8ve quarter-4th decatonic	half-8ve quarter-12th
9L 1s	(P8, P4/2) [10]	quarter-4th decatonic	

The pentatonic MOS scales don't include fifth-split pergens, because a pentatonic genchain has only 4 steps, and can only divide a multigen into quarters. It would be possible to include pergens with a multigen which isn't actually generated. For example, 3L 2s using the sensei generator would be (P8, WWP5/7) [5]. The rationale would be that two sensei generators equals 5/3, in effect a (P8, (5/3)/2) pseudo-pergen.

Some MOS scales are better understood using a pergen with a nonstandard prime subgroup. For example, 6L 1s can be roulette [7], with a 2.5.7 pergen (P8, (5/4)/2) = (P8, M3/2) = (P8, M2), where 5·G = A6 = 7/4.

Pergens and EDOs

Pergens have much in common with edos. Pergens of rank-2 assume only primes 2 and 3, edos assume only prime 2. There are an infinite number of edos and pergens, but only a few dozen of either have been explored.

Just as edos are said to support temperaments, they can support pergens. If a temperament is supported, its pergen is too, and vice versa. An edo can't suppoprt a pergen if the split is impossible. For example, all odd-numbered edos are incompatible with half-octave pergens. A pergen is somewhat unsupported by an edo if the period and generator can only generate a subset of the edo. For example, (P8, P5) is somewhat unsupported by 15edo, because any chain-of-5ths scale could only make a 5-edo subset.

How many pergens are fully supported by a given edo? Surprisingly, an infinite number! There are infinitely many possible multigens, each one divisible by its keyspan. For example, 12edo supports, among other pergens, this series: (P8, P5/7), (P8, P12/19), (P8, WWP5/31),... (P8, (i-1,1)/n), where n = 12i+7.

How many edos support a given pergen? Presumably, an infinite number. For (P8/m, M/n) to be supported by N-edo, N must be a multiple of m, and k must be divisible by n, where k is the multigen's N-edo keyspan. To be fully supported, N/m and k/n must be coprime.

Given an edo, a period, and a generator, what is the pergen? There is usually more than one right answer. For 10edo with P = 5\10 and G = 2\10, it could be either (P8/2, P4/2) or (P8/2, P5/3). Every coprime period/generator pair results in a valid pergen. It isn't yet known if there are period/generator pairs that require a true double pergen, or if all such pairs can result from either a false double or single-split pergen.

EDOs Supporting A Pergen

This table lists all pergens up to quarter-splits, with all edos that support them. Partial support is indicated with an asterisk. The generator's keyspan depends on the multigen's keyspan, and thus on the 5th's keyspan. The latter is occasionally ambiguous, as in 13-edo and 18-edo. Both of these edos are incompatible with heptatonic notation, and 13edo's half-5th pergen is actually notated as a half-upfifth. 13b-edo and 18b-edo are listed as well. 6-edo, 11-edo and 23-edo could also be considered ambiguous.

pergen		supporting edos (12-31 only)
(P8, P5)	unsplit	12, 13, 13b, 14*, 15*, 16, 17, 18, 18b*, 19, 20*, 21*, 22, 23, 24*, 25*, 26, 27, 28*, 29, 30*, 31
halves		
(P8/2, P5)	half-8ve	12, 14, 16, 18, 18b, 20*, 22, 24*, 26, 28*, 30*
(P8, P4/2)	half-4th	13b, 14, 15*, 18b*, 19, 20*, 23, 24, 25*, 28*, 29, 30*
(P8, P5/2)	half-5th	13, 14*, 17, 18b, 20*, 21*, 24, 27, 28*, 30*, 31
(P8/2, P4/2)	half-everything	14, 18b, 20*, 24, 28*, 30*
thirds		
(P8/3, P5)	third-8ve	12, 15, 18, 18b*, 21, 24*, 27, 30*
(P8, P4/3)	third-4th	13b, 14*, 15, 21*, 22, 28*, 29, 30*
(P8, P5/3)	third-5th	15*, 16, 20*, 21, 25*, 26, 30*, 31
(P8, P11/3)	third-11th	13, 15, 17, 21, 23, 30*
(P8/3, P4/2)	third-8ve, half-4th	15, 18b*, 24, 30*
(P8/3, P5/2)	third-8ve, half-5th	18b, 21, 24, 27, 30
(P8/2, P4/3)	half-8ve, third-4th	14, 22, 28*, 30
(P8/2, P5/3)	half-8ve, third-5th	16, 20*, 26, 30*
(P8/2, P11/3)	half-8ve, third-11th	19, 30
(P8/3, P4/3)	third-everything	15, 21, 30*
quarters		
(P8/4, P5)	quarter-8ve	12, 16, 20, 24*, 28
(P8, P4/4)	quarter-4th	18b*, 19, 20*, 28, 29, 30*
(P8, P5/4)	quarter-5th	13, 14*, 20, 21*, 27, 28*
(P8, P11/4)	quarter-11th	14, 17, 20, 28*, 31

(P8, P12/4)	quarter-12th	13b, 15*, 18b, 20*, 23, 25*, 28, 30*
(P8/4, P4/2)	quarter-8ve, half-4th	20, 24, 28
(P8/2, M2/4)	half-8ve, quarter-tone	18, 20, 22, 24, 26, 28
(P8/2, P4/4)	half-8ve, quarter-4th	18b, 20*, 28, 30*
(P8/2, P5/4)	half-8ve, quarter-5th	14, 20, 28*
(P8/4, P4/3)	quarter-8ve, third-4th	28
(P8/4, P5/3)	quarter-8ve, third-5th	16, 20
(P8/4, P11/3)	quarter-8ve, third-11th	
(P8/3, P4/4)	third-8ve, quarter-4th	18b*, 30
(P8/3, P5/4)	third-8ve, quarter-5th	21, 27
(P8/3, P11/4)	third-8ve, quarter-11th	
(P8/3, P12/4)	third-8ve, quarter-12th	15, 18b, 30*
(P8/4, P4/4)	quarter-everything	20, 28

The edos that support the fewest pergens are prime-number edos like 11edo or 13edo. The most "pergen-friendly" edos tend to be ones in which the circle of 5ths doesn't reach every edostep. For example, 24edo supports all half-split pergens, since both P8 and P5 map to an even number of edosteps. 72edo supports all half-splits and all third-splits. 15, 21 and 36 edo support many but not all third-splits (not those with m = 2 or n = 2).

Pergens Within An EDO

See the screenshots in the Supplemental Materials section for a complete list of which pergens are supported by 12, 15 and 19 edo. The list includes multiple pergens per period/generator combination. The next table shows only the simplest pergen for each combination. Combinations are excluded if the period and generator are not coprime, because the scale is contained in a smaller edo. For example, 15edo has no unsplit pergen, because those scales are contained in 5edo. Periods of 1 or 2 edosteps are excluded as too trivial, because the scale is the entire edo. Every step of the edo appears in the genchains, even if the chains are only one step long.

To find the pergen, find the edo, then find the row that corresponds to the period, then find the column that corresponds to the generator.

EDO	Period	Generator in edosteps										
	in edosteps	1	2	3	4	5	6	7	8	9	10	11
5	5 = P8	P4/2	P5									
6	6 = P8	P4/2	-									
"	3 = P8/2	P5										
7	7 = P8	P4/3	P5/2	P5								
8	8 = P8	P4/3	-	P5								
"	4 = P8/2	P5	-									
9	9 = P8	P4/4	P4/2	-	P5							
"	3 = P8/3	P5										
10	10 = P8	P4/4	-	P5/2	-							
"	5 = P8/2	P5	P4/2									
11	11 = P8	P4/5	P5/3	P5/2	P11/4	P5						
12	12 = P8	P4/5	-	-	-	P5						
"	6 = P8/2	P5	-	-								
"	4 = P8/3	P5	-									
"	3 = P8/4	P5										
13b	13 = P8	P4/6	P4/3	P4/2	P12/5	P12/4	P5					
14	14 = P8	P4/6	-	P4/2	-	P11/4	-					
"	7 = P8/2	P5	P4/3	P4/2								
15	15 = P8	P4/6	P4/3	-	P12/6	-	-	P11/3				
"	5 = P8/3	P5	P4/2									
"	3 = P8/5	P4/3										
16	16 = P8	P4/7	-	P5/3	-	P12/5	-	P5				
"	8 = P8/2	P5	-	P5/3	-							
"	4 = P8/4	P5	-									
17	17 = P8	P4/7	P5/5	P11/8	P11/6	P5/2	P11/4	P5	P11/3			
18b	18 = P8	P4/8	-	-	-	P5/2	-	P12/4	-			
"	9 = P8/2	P5	P4/4	-	P4/2							
"	6 = P8/3	P5/2	-	-								
"	3 = P8/6	P5										
19	19 = P8	P4/8	P4/4	P11/9	P4/2	P12/6	P12/5	WWP5/7	P5	P11/3		
20	20 = P8	P4/8	-	P5/4	-	-	-	P11/4	-	W3P5/8		
"	10 = P8/2	M2/4	-	P5/4	-	-						
"	5 = P8/4	P4/2	P5									
"	4 = P8/5	P5/4	-									

21	**21 = P8**	P4/9	P5/6	-	P5/3	P11/6	-	-	W3P4/9	-	P11/3	
"	**7 = P8/3**	P5/2	P5	P4/3								
"	**3 = P8/7**	P5/3										
22	**22 = P8**	P4/9	-	P4/3	-	P12/7	-	P12/5	-	P5	-	
"	**11 = P8/2**	M2/4	P5	P4/3	P12/5	P12/7						
23	**23 = P8**	P4/10	P4/5	P11/11	P12/9	P4/2	P12/6	WWP4/8	WWP4/7	P12/4	P5	P11/3
24	**24 = P8**	P4/10	-	-	-	P4/2	-	P5/2	-	-	-	W4P5/10
"	**12 = P8/2**	M2/4	-	-	-	P4/2						
"	**8 = P8/3**	P5/2	-	P4/2								
"	**6 = P8/4**	P4/2	-	-								
"	**4 = P8/6**	P4/2	-									
"	**3 = P8/8**	P5										
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**

EDO-pair names

Just as a pair of edos and a prime subgroup can specify a rank-2 temperament, a pair of edos can specify a rank-2 pergen. For N-edo & N'-edo, m = GCD (N,N'). The period P equals both (N/m)\N and (N'/m)\N'. For example, for 12edo and 16edo, m = 4, and the period is both 3\12 and 4\16.

For each edo, find the nearest **edomapping** (patent val) for the 2.3 subgroup. If the edo has a "b" wart, e.g. 13b-edo, use the second-nearest edomapping. Form a 2x2 matrix from these edomappings. Let d be the determinant of this matrix. If d = m, -m or 0, the generator is the 5th, and the pergen is simply (P8/m, P5).

For example, 12edo's 3-limit edomapping is (12, 19), and 16edo's is (16, 25). The determinant of [(12 19) (16 25)] is -4, thus the pergen for 12edo and 16edo is (P8/4, P5). To make the calculations easier, octave-reduced edomappings can be used, which indicate the number of edosteps that 3/2 maps to, not 3/1. For 12 and 16, we have [(12 7) (16 9)], and d is again -4.

If |d| ≠ m or 0, P5 is not the generator, and we must find the multigen. Take the ratio of the two edos N/N' and reduce it by m. In the scale tree, let g/g' be the smallest ancestor of this ratio. The generator G maps to both g\N and g'\N'. The two edos come closest to coinciding here.

For example, for 7-edo and 17-edo, m = 1 but d = 2, so G ≠ P5. The smallest ancestor of 7/17 is 2/5. G maps to both 2\7 and 5\17, which are both neutral 3rds. Using the other ancestor always gives the octave inverses, in this case 5/12 giving 5\7 and 12\17, which are neutral 6ths. 2\7 = 343¢ and 5\17 = 353¢, and their difference is only 1\N", where N" = LCM (N, N'). This 10¢ difference is the least difference between any 7edo note and any 17edo note, except that the two neutral 6ths differ by the same 10¢, and of course some note pairs coincide exactly, such as 0\7 and 0\17.

Next we must find a comma that both edos temper out, and find the pergen from that comma. To do this, extend the edomappings to three entries. Rather than finding the mapping of 5/4 or 7/4, we use the generator we found from the scale tree, without concerning ourselves about which ratio it corresponds to, in keeping with the higher-prime-agnostic nature of pergens. Treating the two edomappings as 3-D vectors, take the cross product of them, whch makes a new vector which is perpendicular to both. Thus the dot product of this new vector with either edomapping is zero. Treating this new vector as a monzo, since the dot product is zero, both edos map this monzo to zero edosteps. In other words, they temper out this monzo, and this monzo is the comma we're looking for. If the comma is (a, b, c), then a·P8 + b·P5 + c·G = 0, and G = (b-a, -b) / c.

For example, for 7-edo and 17-edo, the edomappings are (7, 4, 2) and (17, 10, 5). Their cross product is (0, -1, 2). This comma equates two generators with a 5th. The pergen is obviously (P8, P5/2).

To verify the validity of this approach, one can find a specific JI ratio that maps to both 2\7 and 5\17 and use it to construct a comma. The ratio must contain only one higher prime, and must have color depth 1. If desired, a ratio of the form p/t can always be found, where p is a higher prime and t is a power of two. Any ratio between 3\14 and 5\14 maps to 2\7, and any ratio between 9\34 and 11\34 maps to 5\17. (The formula is (2n±1)/2d.) This gives us the ranges 257-429¢ and 318-388¢. Thus 5/4 barely works. The comma is (0, -1, 2) dot (2/1, 3/2, 5/4) = 25/24. 11/9 also works, it yields 243/242.

If the two edos have the same 5th, such as 12edo and 24edo do, the 5th is some multiple of the period, and the pergen is a Blackwood-like pergen.

The closer two edos are in the scale tree, the simpler the pergen they make. Examples::

	12-edo	**13b-edo**	**14-edo**	**15-edo**	**16-edo**	**17-edo**	**18b-edo**	**19-edo**	**20-edo**
13b-edo	(P8, P5/7)								
14-edo	(P8/2, P5)	(P8, P4/6)							
15-edo	(P8/3, P5)	(P8, W5P5/12)	(P8, P4/6)						
16-edo	(P8/4, P5)	(P8, P12/5)	(P8/2, P5)	(P8, P5/9)					
17-edo	(P8, P5)	(P8, WWP5/11)	(P8, P11/4)	(P8, P11/3)	(P8/2, P4/7)				
18b-edo	(P8/6, P5)	(P8, P12/4)	(P8/2, P4/2)	(P8/3, P12/4)	(P8/2, P5)	(P8, P5/10)			
19-edo	(P8, P5)	(P8, P12/10)	(P8, P4/2)	(P8, P12/6)	(P8, P12/5)	(P8, P11/3)	(P8, P4/8)		
20-edo	(P8/4, P5)	(P8, WWP4/16)	(P8/2, P5/4)	(P8/5, ^1)	(P8/4, P5/3)	(P8, P11/4)	(P8/2, P4/8)	(P8, P4/8)	
21-edo	(P8/3, P5)	(P8, W3P4/9)	(P8/7, ^1)	(P8/3, P4/3)	(P8, P5/3)	(P8, P11/6)	(P8/3, P5/2)	(P8, P11/3)	(P8, P5/12)
22-edo	(P8/2, P5)	(P8, W3P4/15)	(P8/2, P4/3)	(P8, P4/3)	(P8/2, P12/5)	(P8, P5)	(P8/2, P12/7)	(P8, P12/5)	(P8/2, M2/4)
23-edo	(P8, P4/5)	(P8, WWP4/8)	(P8, P4/2)	(P8, P12/12)	(P8, P5)	(P8, P12/9)	(P8, P12/4)	(P8, P12/6)	(P8, W5P5/16)
	(P8/12, ^1)		(P8/2, P4/2)	(P8/3, P4/2)	(P8/8, P5)	(P8, P5/2)	(P8/6, P4/2)	(P8, P4/2)	(P8/4, P4/2)

24-edo		(P8, W6P4/14)								

A specific pergen can be converted to an edo pair by finding the range of its generator cents in the arbitrary generator table, looking up that cents in the scale tree, and finding a conveniently-sized parent-child pair of edos in that range. For example, half-5th has a generator in the 320-360¢ range, and that part of the scale tree has among others 2\7, 3\10 and 5\17. Any two of those three edos defines (P8, P5/2).

Array Keyboards (unfinished)

Array keyboards have a 2-dimensional layout of keys, and are very appropriate for rank-2 tunings. A good layout can be found from the tuning's pergen. First find an edo N-edo that is compatible with the pergen, then arrange the keys in N columns to the 8ve, with each row usually containing the multigen interval. The unsplit pergen in 7 columns:

D#							
D	E	F#	G#	A#			
Db	Eb	F	G	A	B	C#	D#
			Gb	Ab	Bb	C	D
							Db

Higher notes are at the top of each column. The rows would actually be angled so that the two D's are at the same horizontal level. The vertical steps are A1 and the horizontal steps are M2, and the keyboard is defined as 7(A1, M2).

The third-4th keyboard is 7(A1/3 = ^1 = vvA1, P4/3 = vM2 = ^^m2).

D#v	E^	F#	G#v	A^	B	C#v	D^
D^	E	F#v	G^	A	Bv	C^	D
D	Ev	F^	G	Av	B^	C	Dv
Dv	Eb^	F	Gv	Ab^	Bb	Cv	Db^

Hypothesis: Let the 5th's keyspan (i.e. column-span) be F. In order for the keyboard to have the pitches in order, the fifth must fall between the two Stern-Brocot ancestors of F\N (simplified if possible). For example, an 8-column keyboard has F = 5, the ancestors of 5\8 are 3\5 and 2\3, and the 5th must be between 720¢ and 800¢. Thus the most musically useful N values are 5, 7, 10, 12 and 14.
(more to come)

Supplemental materials

Notation guide PDF

This PDF is a rank-2 notation guide that shows the full lattice for the first 32 pergens, up through the quarter-splits block. It includes alternate enharmonics for many pergens.
tallkite.com/misc_files/notation guide for rank-2 pergens.pdf

Screenshots of the first 2 pages:

RANK-2 PERGENS AND THEIR NOTATIONS

P = Period
G = Generator
E = Enharmonic Interval

^ v = up or down, / \ = high or low
c = cents difference of the 5th from 12-edo (700¢)
Adding E to P or G makes an equivalent period/generator

(P8, P5) unsplit

= 100¢ + 7c

600¢	100¢	800¢	**300¢**	**1000¢**	**500¢**	**0¢**	**700¢**	**200¢**	**900¢**	400¢	1100¢	600¢
-6c	-5c	-4c	-3c	-2c	-c		+c	+2c	+3c	+4c	+5c	+6c

P = P8 = 1200¢ G = P5 = 700¢ + c

d5	m2	m6	**m3**	**m7**	**P4**	**P1**	**P5**	**M2**	**M6**	M3	M7	A4

Ab	Eb	Bb	**F**	**C**	**G**	**D**	**A**	**E**	**B**	F#	C#	G#

(P8/2, P5) half-octave

(2 possible notations)

0¢	700¢	200¢	900¢	400¢	1100¢	600¢	100¢	800¢	300¢	1000¢	500¢	0¢
600¢	100¢	800¢	**300¢**	**1000¢**	**500¢**	**0¢**	**700¢**	**200¢**	**900¢**	400¢	1100¢	600¢
0¢	700¢	200¢	900¢	400¢	1100¢	600¢	100¢	800¢	300¢	1000¢	500¢	0¢
-6c	-5c	-4c	-3c	-2c	-c		+c	+2c	+3c	+4c	+5c	+6c

#1 P = vA4 = 600¢ G = P5 = 700¢ + c E = ^^d2 = 0¢ C^^ = Dbb ^ = 6c

v1	v5	vM2	vM6	vM3	vM8	vA4	vA1	vA5	vA2	vA6	vA3	vA7
d5	m2	m6	**m3**	**m7**	**P4**	**P1**	**P5**	**M2**	**M6**	M3	M7	A4
^d8	^d6	^d3	^d7	^d4	^d1	^d5	^m2	^m6	^m3	^m7	^4	^1

Dv	Av	Ev	Bv	F#v	D#v	G#v	D#v	A#v	E#v	B#v	F##v	C##v
Ab	Eb	Bb	**F**	**C**	**G**	**D**	**A**	**E**	**B**	F#	C#	G#
Dbb^	Bbb^	Fb^	Cb^	Gb^	Db^	Ab^	Eb^	Bb^	F^	C^	G^	D^

#2 P = ^4 = 600¢ G = P5 = 700¢ + c E = vvM2 = 0¢ C^^ = D ^ = 100¢ + c

^d1	^d5	^m2	^m6	^m3	^m8	^4	^1	^5	^M2	^M6	^M3	^M7
d5	m2	m6	**m3**	**m7**	**P4**	**P1**	**P5**	**M2**	**M6**	M3	M7	A4
vm8	vm6	vm3	vm7	v4	v1	v5	vM2	vM6	vM3	vM7	vA4	vA1

Db^	Ab^	Eb^	Bb^	F^	D^	G^	D^	A^	E^	B^	F#^	C#^
Ab	Eb	Bb	**F**	**C**	**G**	**D**	**A**	**E**	**B**	F#	C#	G#
Dbv	Bbv	Fv	Cv	Gv	Dv	Av	Ev	Bv	F#v	C#v	G#v	D#v

(P8, P4/2) **half-fourth** (2 possible notations)

300¢	50¢	**1000¢**	750¢	**500¢**	250¢	**0¢**	950¢	**700¢**	450¢	**200¢**	1150¢	**900¢**
-3c	-2.5c	-2c	-1.5c	-c	-0.5c		+0.5c	+c	+1.5c	+2c	+2.5c	+3c

#1 P = P8 = 1200¢ G = ^M2 = 250¢ - c/2 E = vvm2 = 0¢ C^^ =Db ^ = 50¢ - 2.5c

m3	vm2 ^1	**m7**	vm6 ^5	**P4**	vm3 ^M2	**P1**	vm7 ^M6	**P5**	v4 ^M3	**M2**	v8 ^M7	**M6**

F	Ebv D^	**C**	Bbv A^	**G**	Fv E^	**D**	Cv B^	**A**	Gv F#^	**E**	Dv C#^	**B**

#2 P = P8 = 1200¢ G = vA2 = 250¢ - c/2 E = ^^dd2 = 0¢ C^^ = Dbb ^1 = 50¢ + 9.5c

m3	^d2 vA1	**m7**	^d6 vA5	**P4**	^d3 vA2	**P1**	^d7 vA6	**P5**	^d4 vA3	**M2**	^d8 vA7	**M6**

F	Ebb^ D#v	**C**	Bbb^ A#v	**G**	Fb^ E#v	**D**	Cb^ B#v	**A**	Gb^ F##v	**E**	Db^ C##v	**B**

(P8, P5/2) **half-fifth**

300¢	650¢	**1000¢**	150¢	**500¢**	850¢	**0¢**	350¢	**700¢**	1050¢	**200¢**	550¢	**900¢**
-3c	-2.5c	-2c	-1.5c	-c	-0.5c		+0.5c	+c	+1.5c	+2c	+2.5c	+3c

P = P8 = 1200¢ G = ^m3 = 350¢ + c/2 E = vvA1 = 0¢ C^^ = C# ^ = 50¢ + 3.5c

m3	v5 ^d5	**m7**	vM2 ^m2	**P4**	vM6 ^m6	**P1**	vM3 ^m3	**P5**	vM7 ^m7	**M2**	vA4 ^4	**M6**

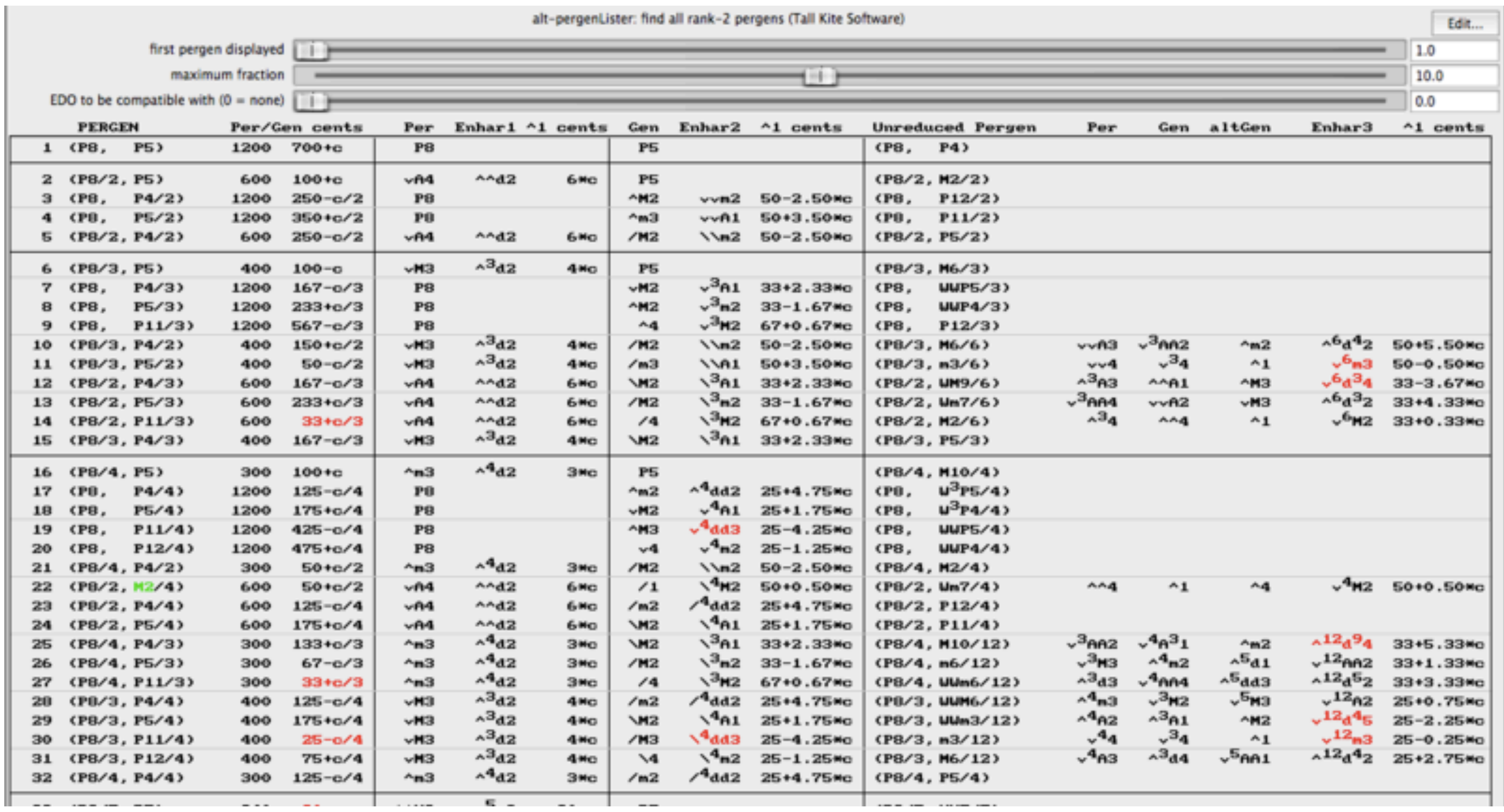
F	Av Ab^	**C**	Ev Eb^	**G**	Bv Bb^	**D**	F#v F^	**A**	C#v C^	**E**	G#v G^	**B**

alt-pergenLister

Alt-pergenLister lists out thousands of rank-2 pergens, and suggests periods, generators and enharmonics for each one. Alternate enharmonics are not listed, but single-pair notation for false-double pergens is. It can also list only those pergens supported by a specific edo or edo pair. Written in Jesusonic, runs inside Reaper.
http://www.tallkite.com/misc_files/alt-pergenLister.zip

The first section (PERGEN and Per/Gen cents) describes each pergen without regard to notational issues. The period and generator's cents are given, assuming a 5th of 700¢ + c. The generator is reduced, e.g. (P8/2, P5) has a generator of 100¢ + c, not 700¢ + c. The next two sections show a possible notation for P and G. The last section shows the unreduced pergen, and for false doubles, a possible single-pair notation. Horizontal lines group the pergens into blocks (half-splits, third-splits, etc). Red indicates problems. Generators of 50¢ or less are in red. Enharmonics of a 3rd or more are in red.

Screenshots of the first 69 pergens:



alt-pergenLister: find all rank-2 pergens (Tall Kite Software)

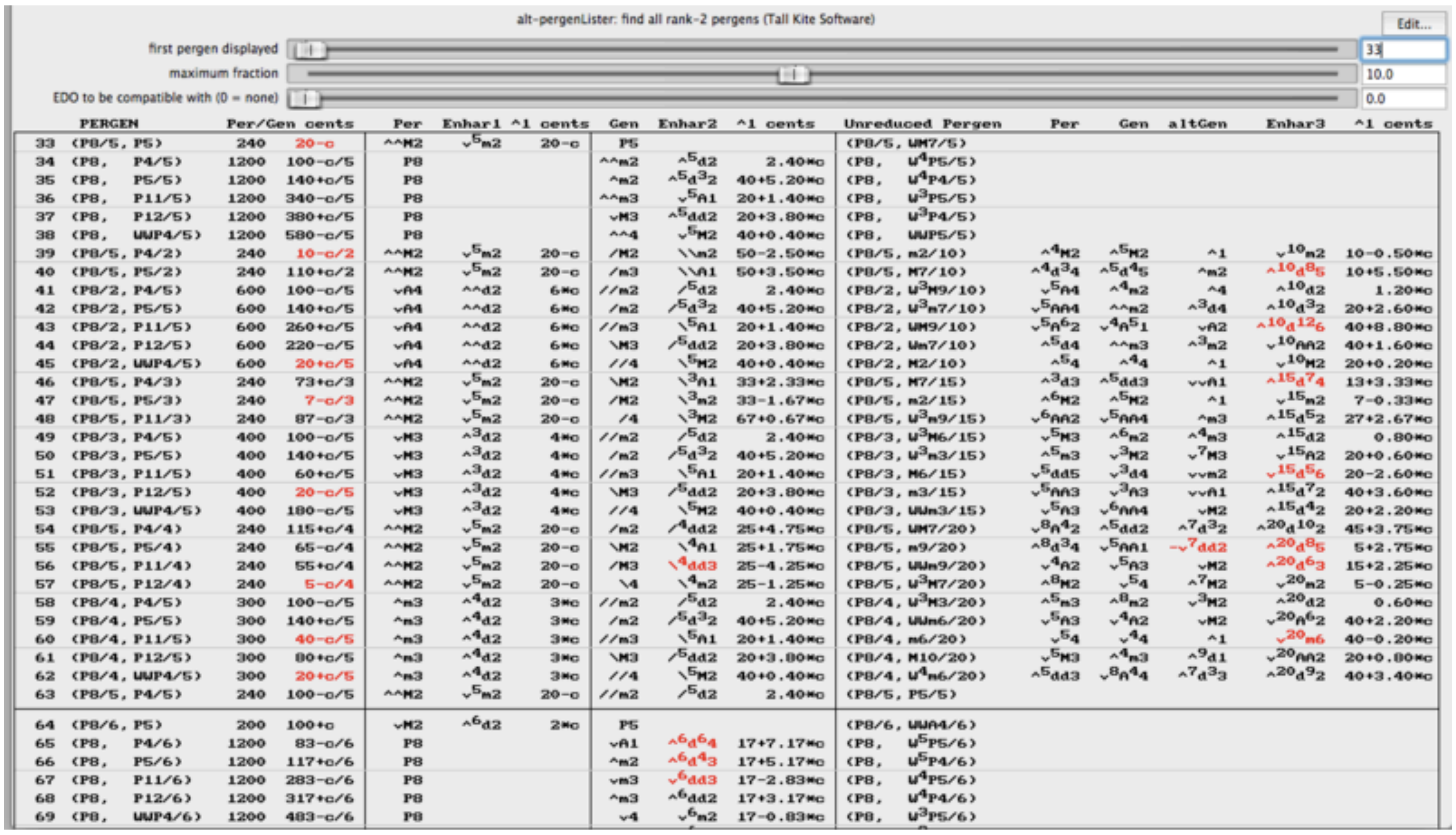
Edit...

first pergen displayed 1.0

maximum fraction 10.0

EDO to be compatible with (0 = none) 0.0

	PERGEN	Per/Gen cents		Per	Enhar1	^1 cents	Gen	Enhar2	^1 cents	Unreduced Pergen	Per	Gen	altGen	Enhar3	^1 cents
1	(P8, P5)	1200	700+c	P8			P5			(P8, P4)					
2	(P8/2, P5)	600	100+c	vA4	^^d2	6*c	P5			(P8/2, M2/2)					
3	(P8, P4/2)	1200	250-c/2	P8			^M2	vvm2	50-2.50*c	(P8, P12/2)					
4	(P8, P5/2)	1200	350+c/2	P8			^m3	vvA1	50+3.50*c	(P8, P11/2)					
5	(P8/2, P4/2)	600	250-c/2	vA4	^^d2	6*c	/M2	\\m2	50-2.50*c	(P8/2, P5/2)					
6	(P8/3, P5)	400	100-c	vM3	^3d2	4*c	P5			(P8/3, M6/3)					
7	(P8, P4/3)	1200	167-c/3	P8			vM2	v3A1	33+2.33*c	(P8, WWP5/3)					
8	(P8, P5/3)	1200	233+c/3	P8			^M2	v3m2	33-1.67*c	(P8, WWP4/3)					
9	(P8, P11/3)	1200	567-c/3	P8			^4	v3M2	67+0.67*c	(P8, P12/3)					
10	(P8/3, P4/2)	400	150+c/2	vM3	^3d2	4*c	/M2	\\m2	50-2.50*c	(P8/3, M6/6)	vvA3	v3AA2	^m2	^6d42	50+5.50*c
11	(P8/3, P5/2)	400	50-c/2	vM3	^3d2	4*c	/m3	\\A1	50+3.50*c	(P8/3, m3/6)	vv4	v34	^1	v6m3	50-0.50*c
12	(P8/2, P4/3)	600	167-c/3	vA4	^^d2	6*c	\M2	\3A1	33+2.33*c	(P8/2, WM9/6)	^3A3	^^A1	^M3	v6d34	33-3.67*c
13	(P8/2, P5/3)	600	233+c/3	vA4	^^d2	6*c	/M2	\3m2	33-1.67*c	(P8/2, Wm7/6)	v3AA4	vvA2	vM3	^6d32	33+4.33*c
14	(P8/2, P11/3)	600	33+c/3	vA4	^^d2	6*c	/4	\3M2	67+0.67*c	(P8/2, M2/6)	^34	^^4	^1	v6M2	33+0.33*c
15	(P8/3, P4/3)	400	167-c/3	vM3	^3d2	4*c	\M2	\3A1	33+2.33*c	(P8/3, P5/3)					
16	(P8/4, P5)	300	100+c	^m3	^4d2	3*c	P5			(P8/4, M10/4)					
17	(P8, P4/4)	1200	125-c/4	P8			^m2	^4dd2	25+4.75*c	(P8, W3P5/4)					
18	(P8, P5/4)	1200	175+c/4	P8			vM2	v4A1	25+1.75*c	(P8, W3P4/4)					
19	(P8, P11/4)	1200	425-c/4	P8			^M3	v4dd3	25-4.25*c	(P8, WWP5/4)					
20	(P8, P12/4)	1200	475+c/4	P8			v4	v4m2	25-1.25*c	(P8, WWP4/4)					
21	(P8/4, P4/2)	300	50+c/2	^m3	^4d2	3*c	/M2	\\m2	50-2.50*c	(P8/4, M2/4)					
22	(P8/2, M2/4)	600	50+c/2	vA4	^^d2	6*c	/1	\4M2	50+0.50*c	(P8/2, Wm7/4)	^^4	^1	^4	v4M2	50+0.50*c
23	(P8/2, P4/4)	600	125-c/4	vA4	^^d2	6*c	/m2	/4dd2	25+4.75*c	(P8/2, P12/4)					
24	(P8/2, P5/4)	600	175+c/4	vA4	^^d2	6*c	\M2	\4A1	25+1.75*c	(P8/2, P11/4)					
25	(P8/4, P4/3)	300	133+c/3	^m3	^4d2	3*c	\M2	\3A1	33+2.33*c	(P8/4, M10/12)	v3AA2	v4A31	^m2	^12d94	33+5.33*c
26	(P8/4, P5/3)	300	67-c/3	^m3	^4d2	3*c	/M2	\3m2	33-1.67*c	(P8/4, m6/12)	v3M3	^4m2	^5d1	v12AA2	33+1.33*c
27	(P8/4, P11/3)	300	33+c/3	^m3	^4d2	3*c	/4	\3M2	67+0.67*c	(P8/4, WWm6/12)	^3d3	v4AA4	^5dd3	^12d52	33+3.33*c
28	(P8/3, P4/4)	400	125-c/4	vM3	^3d2	4*c	/m2	/4dd2	25+4.75*c	(P8/3, WWM6/12)	^4m3	v3M2	v5M3	v12A2	25+0.75*c
29	(P8/3, P5/4)	400	175+c/4	vM3	^3d2	4*c	\M2	\4A1	25+1.75*c	(P8/3, WWm3/12)	^4A2	^3A1	^M2	v12d46	25-2.25*c
30	(P8/3, P11/4)	400	25-c/4	vM3	^3d2	4*c	/M3	\4dd3	25-4.25*c	(P8/3, m3/12)	v44	v34	^1	v12m3	25-0.25*c
31	(P8/3, P12/4)	400	75+c/4	vM3	^3d2	4*c	\4	\4m2	25-1.25*c	(P8/3, M6/12)	v4A3	^3d4	v5AA1	^12d42	25+2.75*c
32	(P8/4, P4/4)	300	125-c/4	^m3	^4d2	3*c	/m2	/4dd2	25+4.75*c	(P8/4, P5/4)					



alt-pergenLister: find all rank-2 pergens (Tall Kite Software)

Edit...

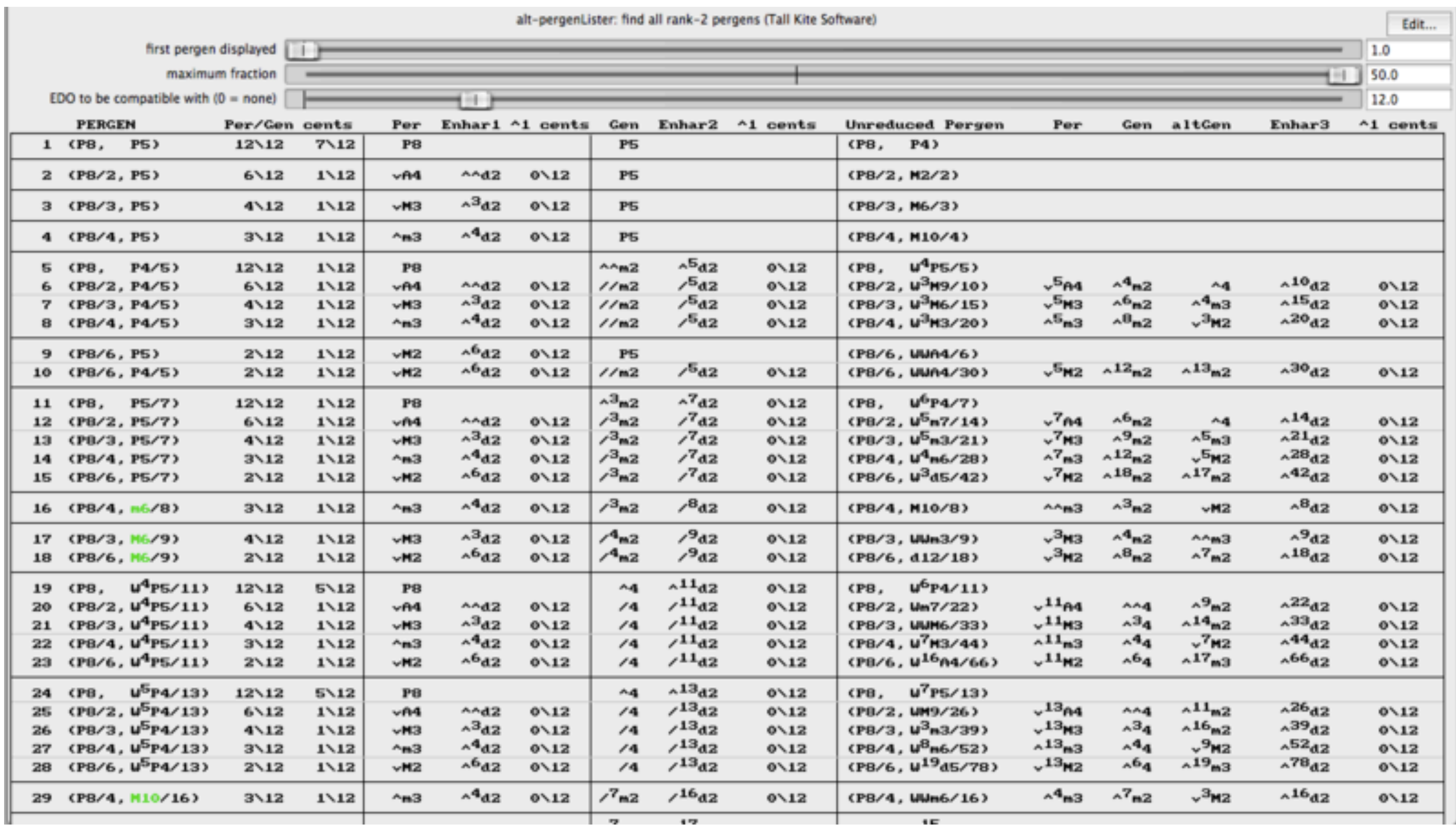
first pergen displayed 33

maximum fraction 10.0

EDO to be compatible with (0 = none) 0.0

	PERGEN	Per/Gen cents		Per	Enhar1	^1 cents	Gen	Enhar2	^1 cents	Unreduced Pergen	Per	Gen	altGen	Enhar3	^1 cents
33	(P8/5, P5)	240	20-c	^^M2	v5m2	20-c	P5			(P8/5, WM7/5)					
34	(P8, P4/5)	1200	100-c/5	P8			^^m2	^5d2	2.40*c	(P8, W4P5/5)					
35	(P8, P5/5)	1200	140+c/5	P8			^m2	^5d32	40+5.20*c	(P8, W4P4/5)					
36	(P8, P11/5)	1200	340-c/5	P8			^^m3	v5A1	20+1.40*c	(P8, W3P5/5)					
37	(P8, P12/5)	1200	380+c/5	P8			vM3	^5dd2	20+3.80*c	(P8, W3P4/5)					
38	(P8, WWP4/5)	1200	580-c/5	P8			^^4	v5M2	40+0.40*c	(P8, WWP5/5)					
39	(P8/5, P4/2)	240	10-c/2	^^M2	v5m2	20-c	/M2	\\m2	50-2.50*c	(P8/5, m2/10)	^4M2	^5M2	^1	v10m2	10-0.50*c
40	(P8/5, P5/2)	240	110+c/2	^^M2	v5m2	20-c	/m3	\\A1	50+3.50*c	(P8/5, M7/10)	^4d34	^5d45	^m2	^10d85	10+5.50*c
41	(P8/2, P4/5)	600	100-c/5	vA4	^^d2	6*c	//m2	/5d2	2.40*c	(P8/2, W3M9/10)	v5A4	^4m2	^4	^10d2	1.20*c
42	(P8/2, P5/5)	600	140+c/5	vA4	^^d2	6*c	/m2	/5d32	40+5.20*c	(P8/2, W3m7/10)	v5AA4	^^m2	^3d4	^10d32	20+2.60*c
43	(P8/2, P11/5)	600	260+c/5	vA4	^^d2	6*c	//m3	\5A1	20+1.40*c	(P8/2, WM9/10)	v5A62	v4A51	vA2	^10d126	40+8.80*c
44	(P8/2, P12/5)	600	220-c/5	vA4	^^d2	6*c	\M3	/5dd2	20+3.80*c	(P8/2, Wm7/10)	^5d4	^^m3	^3m2	v10AA2	40+1.60*c
45	(P8/2, WWP4/5)	600	20+c/5	vA4	^^d2	6*c	//4	\5M2	40+0.40*c	(P8/2, M2/10)	^54	^44	^1	v10M2	20+0.20*c
46	(P8/5, P4/3)	240	73+c/3	^^M2	v5m2	20-c	\M2	\3A1	33+2.33*c	(P8/5, M7/15)	^3d3	^5dd3	vvA1	^15d74	13+3.33*c
47	(P8/5, P5/3)	240	7-c/3	^^M2	v5m2	20-c	/M2	\3m2	33-1.67*c	(P8/5, m2/15)	^6M2	^5M2	^1	v15m2	7-0.33*c
48	(P8/5, P11/3)	240	87-c/3	^^M2	v5m2	20-c	/4	\3M2	67+0.67*c	(P8/5, W3m9/15)	v6AA2	v5AA4	^m3	^15d52	27+2.67*c
49	(P8/3, P4/5)	400	100-c/5	vM3	^3d2	4*c	//m2	/5d2	2.40*c	(P8/3, W3M6/15)	v5M3	^6m2	^4m3	^15d2	0.80*c
50	(P8/3, P5/5)	400	140+c/5	vM3	^3d2	4*c	/m2	/5d32	40+5.20*c	(P8/3, W3m3/15)	^5m3	v3M2	v7M3	v15A2	20+0.60*c
51	(P8/3, P11/5)	400	60+c/5	vM3	^3d2	4*c	//m3	\5A1	20+1.40*c	(P8/3, M6/15)	v5dd5	v3d4	vvm2	v15d56	20-2.60*c
52	(P8/3, P12/5)	400	20-c/5	vM3	^3d2	4*c	\M3	/5dd2	20+3.80*c	(P8/3, m3/15)	v5AA3	v3A3	vvA1	^15d72	40+3.60*c
53	(P8/3, WWP4/5)	400	180-c/5	vM3	^3d2	4*c	//4	\5M2	40+0.40*c	(P8/3, WWm3/15)	v5A3	v6AA4	vM2	^15d42	20+2.20*c
54	(P8/5, P4/4)	240	115+c/4	^^M2	v5m2	20-c	/m2	/4dd2	25+4.75*c	(P8/5, WM7/20)	v8A42	^5dd2	^7d32	^20d102	45+3.75*c
55	(P8/5, P5/4)	240	65-c/4	^^M2	v5m2	20-c	\M2	\4A1	25+1.75*c	(P8/5, m9/20)	^8d34	v5AA1	-v7dd2	^20d85	5+2.75*c
56	(P8/5, P11/4)	240	55+c/4	^^M2	v5m2	20-c	/M3	\4dd3	25-4.25*c	(P8/5, WWm9/20)	v4A2	v5A3	vM2	^20d63	15+2.25*c
57	(P8/5, P12/4)	240	5-c/4	^^M2	v5m2	20-c	\4	\4m2	25-1.25*c	(P8/5, W3M7/20)	^8M2	v54	^7M2	v20m2	5-0.25*c
58	(P8/4, P4/5)	300	100-c/5	^m3	^4d2	3*c	//m2	/5d2	2.40*c	(P8/4, W3M3/20)	^5m3	^8m2	v3M2	^20d2	0.60*c
59	(P8/4, P5/5)	300	140+c/5	^m3	^4d2	3*c	/m2	/5d32	40+5.20*c	(P8/4, WWm6/20)	v5A3	v4A2	vM2	v20A62	40+2.20*c
60	(P8/4, P11/5)	300	40-c/5	^m3	^4d2	3*c	//m3	\5A1	20+1.40*c	(P8/4, m6/20)	v54	v44	^1	v20m6	40-0.20*c
61	(P8/4, P12/5)	300	80+c/5	^m3	^4d2	3*c	\M3	/5dd2	20+3.80*c	(P8/4, M10/20)	v5M3	^4m3	^9d1	v20AA2	20+0.80*c
62	(P8/4, WWP4/5)	300	20+c/5	^m3	^4d2	3*c	//4	\5M2	40+0.40*c	(P8/4, W4m6/20)	^5dd3	v8A44	^7d33	^20d92	40+3.40*c
63	(P8/5, P4/5)	240	100-c/5	^^M2	v5m2	20-c	//m2	/5d2	2.40*c	(P8/5, P5/5)					
64	(P8/6, P5)	200	100+c	vM2	^6d2	2*c	P5			(P8/6, WWA4/6)					
65	(P8, P4/6)	1200	83-c/6	P8			vA1	^6d64	17+7.17*c	(P8, W5P5/6)					
66	(P8, P5/6)	1200	117+c/6	P8			^m2	^6d43	17+5.17*c	(P8, W5P4/6)					
67	(P8, P11/6)	1200	283-c/6	P8			vm3	v6dd3	17-2.83*c	(P8, W4P5/6)					
68	(P8, P12/6)	1200	317+c/6	P8			^m3	^6dd2	17+3.17*c	(P8, W4P4/6)					
69	(P8, WWP4/6)	1200	483-c/6	P8			v4	v6m2	17-0.83*c	(P8, W3P5/6)					

The first 29 pergens supported by 12edo:



alt-pergenLister: find all rank-2 pergens (Tall Kite Software)

Edit...

first pergen displayed 1.0

maximum fraction 50.0

EDO to be compatible with (0 = none) 12.0

	PERGEN	Per/Gen cents		Per	Enhar1	^1 cents	Gen	Enhar2	^1 cents	Unreduced Pergen	Per	Gen	altGen	Enhar3	^1 cents
1	(P8, P5)	12\12	7\12	P8			P5			(P8, P4)					
2	(P8/2, P5)	6\12	1\12	vA4	^^d2	0\12	P5			(P8/2, M2/2)					
3	(P8/3, P5)	4\12	1\12	vM3	^3d2	0\12	P5			(P8/3, M6/3)					
4	(P8/4, P5)	3\12	1\12	^m3	^4d2	0\12	P5			(P8/4, M10/4)					
5	(P8, P4/5)	12\12	1\12	P8			^^m2	^5d2	0\12	(P8, W4P5/5)					
6	(P8/2, P4/5)	6\12	1\12	vA4	^^d2	0\12	//m2	/5d2	0\12	(P8/2, W3M9/10)	v5A4	^4m2	^4	^10d2	0\12
7	(P8/3, P4/5)	4\12	1\12	vM3	^3d2	0\12	//m2	/5d2	0\12	(P8/3, W3M6/15)	v5M3	^6m2	^4m3	^15d2	0\12
8	(P8/4, P4/5)	3\12	1\12	^m3	^4d2	0\12	//m2	/5d2	0\12	(P8/4, W3M3/20)	^5m3	^8m2	v3M2	^20d2	0\12
9	(P8/6, P5)	2\12	1\12	vM2	^6d2	0\12	P5			(P8/6, WWA4/6)					
10	(P8/6, P4/5)	2\12	1\12	vM2	^6d2	0\12	//m2	/5d2	0\12	(P8/6, WWA4/30)	v5M2	^12m2	^13m2	^30d2	0\12
11	(P8, P5/7)	12\12	1\12	P8			^3m2	^7d2	0\12	(P8, W6P4/7)					
12	(P8/2, P5/7)	6\12	1\12	vA4	^^d2	0\12	/3m2	/7d2	0\12	(P8/2, W5m7/14)	v7A4	^6m2	^4	^14d2	0\12
13	(P8/3, P5/7)	4\12	1\12	vM3	^3d2	0\12	/3m2	/7d2	0\12	(P8/3, W5m3/21)	v7M3	^9m2	^5m3	^21d2	0\12
14	(P8/4, P5/7)	3\12	1\12	^m3	^4d2	0\12	/3m2	/7d2	0\12	(P8/4, W4m6/28)	^7m3	^12m2	v5M2	^28d2	0\12
15	(P8/6, P5/7)	2\12	1\12	vM2	^6d2	0\12	/3m2	/7d2	0\12	(P8/6, W3d5/42)	v7M2	^18m2	^17m2	^42d2	0\12
16	(P8/4, m6/8)	3\12	1\12	^m3	^4d2	0\12	/3m2	/8d2	0\12	(P8/4, M10/8)	^^m3	^3m2	vM2	^8d2	0\12
17	(P8/3, M6/9)	4\12	1\12	vM3	^3d2	0\12	/4m2	/9d2	0\12	(P8/3, WWm3/9)	v3M3	^4m2	^^m3	^9d2	0\12
18	(P8/6, M6/9)	2\12	1\12	vM2	^6d2	0\12	/4m2	/9d2	0\12	(P8/6, d12/18)	v3M2	^8m2	^7m2	^18d2	0\12
19	(P8, W4P5/11)	12\12	5\12	P8			^4	^11d2	0\12	(P8, W6P4/11)					
20	(P8/2, W4P5/11)	6\12	1\12	vA4	^^d2	0\12	/4	/11d2	0\12	(P8/2, Wm7/22)	v11A4	^^4	^9m2	^22d2	0\12
21	(P8/3, W4P5/11)	4\12	1\12	vM3	^3d2	0\12	/4	/11d2	0\12	(P8/3, WWM6/33)	v11M3	^34	^14m2	^33d2	0\12
22	(P8/4, W4P5/11)	3\12	1\12	^m3	^4d2	0\12	/4	/11d2	0\12	(P8/4, W7M3/44)	^11m3	^44	v7M2	^44d2	0\12
23	(P8/6, W4P5/11)	2\12	1\12	vM2	^6d2	0\12	/4	/11d2	0\12	(P8/6, W16A4/66)	v11M2	^64	^17m3	^66d2	0\12
24	(P8, W5P4/13)	12\12	5\12	P8			^4	^13d2	0\12	(P8, W7P5/13)					
25	(P8/2, W5P4/13)	6\12	1\12	vA4	^^d2	0\12	/4	/13d2	0\12	(P8/2, WM9/26)	v13A4	^^4	^11m2	^26d2	0\12
26	(P8/3, W5P4/13)	4\12	1\12	vM3	^3d2	0\12	/4	/13d2	0\12	(P8/3, W3m3/39)	v13M3	^34	^16m2	^39d2	0\12
27	(P8/4, W5P4/13)	3\12	1\12	^m3	^4d2	0\12	/4	/13d2	0\12	(P8/4, W8m6/52)	^13m3	^44	v9M2	^52d2	0\12
28	(P8/6, W5P4/13)	2\12	1\12	vM2	^6d2	0\12	/4	/13d2	0\12	(P8/6, W19d5/78)	v13M2	^64	^19m3	^78d2	0\12
29	(P8/4, M10/16)	3\12	1\12	^m3	^4d2	0\12	/7m2	/16d2	0\12	(P8/4, WWm6/16)	^4m3	^7m2	v3M2	^16d2	0\12

Some of the pergens supported by 15edo. A red asterisk means partial support.

alt-pergenLister: find all rank-2 pergens (Tall Kite Software)

Edit...

first pergen displayed 1.0
maximum fraction 50.0
EDO to be compatible with (0 = none) 15.0

	PERGEN	Per/Gen cents		Per	Enhar1	^1 cents	Gen	Enhar2	^1 cents	Unreduced Pergen	Per	Gen	altGen	Enhar3	^1 cents
1	(P8, P5)*	15\15	9\15	P8			P5			(P8, P4)					
2	(P8, P4/2)*	15\15	3\15	P8			^M2	vvm2	0\15	(P8, P12/2)					
3	(P8/3, P5)	5\15	1\15	vM3	^^{3}d2	1\15	P5			(P8/3, M6/3)					
4	(P8, P4/3)	15\15	2\15	P8			vM2	v^{3}A1	1\15	(P8, WWP5/3)					
5	(P8, P5/3)*	15\15	3\15	P8			^M2	v^{3}m2	0\15	(P8, WWP4/3)					
6	(P8, P11/3)	15\15	7\15	P8			^4	v^{3}M2	1\15	(P8, P12/3)					
7	(P8/3, P4/2)	5\15	2\15	vM3	^^{3}d2	1\15	/M2	\\m2	0\15	(P8/3, M6/6)	vvA3	v^{3}AA2	^m2	^^{6}d^{4}2	2\15
8	(P8/3, P4/3)	5\15	2\15	vM3	^^{3}d2	1\15	\M2	\$^{3}$A1	1\15	(P8/3, P5/3)					
9	(P8, P12/4)*	15\15	6\15	P8			v4	v^{4}m2	0\15	(P8, WWP4/4)					
10	(P8/3, P12/4)	5\15	1\15	vM3	^^{3}d2	1\15	\4	\$^{4}$m2	0\15	(P8/3, M6/12)	v^{4}A3	^^{3}d4	v^{5}AA1	^^{12}d^{4}2	1\15
11	(P8/5, P5)*	3\15	0\15	^^M2	v^{5}m2	0\15	P5			(P8/5, WM7/5)					
12	(P8/5, P4/2)*	3\15	0\15	^^M2	v^{5}m2	0\15	/M2	\\m2	0\15	(P8/5, m2/10)	^^{4}M2	^^{5}M2	^1	v^{10}m2	0\15
13	(P8/5, P4/3)	3\15	1\15	^^M2	v^{5}m2	0\15	\M2	\$^{3}$A1	1\15	(P8/5, M7/15)	^^{3}d3	^^{5}dd3	vvA1	^^{15}d^{7}4	1\15
14	(P8/5, P5/3)*	3\15	0\15	^^M2	v^{5}m2	0\15	/M2	\$^{3}$m2	0\15	(P8/5, m2/15)	^^{6}M2	^^{5}M2	^1	v^{15}m2	0\15
15	(P8/5, P11/3)	3\15	1\15	^^M2	v^{5}m2	0\15	/4	\$^{3}$M2	1\15	(P8/5, W^{3}m9/15)	v^{6}AA2	v^{5}AA4	^m3	^^{15}d^{5}2	1\15
16	(P8/5, P12/4)*	3\15	0\15	^^M2	v^{5}m2	0\15	\4	\$^{4}$m2	0\15	(P8/5, W^{3}M7/20)	^^{8}M2	v^{5}4	^^{7}M2	v^{20}m2	0\15
17	(P8, P4/6)	15\15	1\15	P8			vA1	^^{6}d^{6}4	2\15	(P8, W^{5}P5/6)					
18	(P8, P12/6)	15\15	4\15	P8			^m3	^^{6}dd2	1\15	(P8, W^{4}P4/6)					
19	(P8, WWP4/6)*	15\15	6\15	P8			v4	v^{6}m2	0\15	(P8, W^{3}P5/6)					
20	(P8/3, P4/6)	5\15	1\15	vM3	^^{3}d2	1\15	\A1	/^{6}d^{6}4	2\15	(P8/3, P12/6)					
21	(P8/5, P4/6)	3\15	1\15	^^M2	v^{5}m2	0\15	\A1	/^{6}d^{6}4	2\15	(P8/5, W^{3}M7/30)	v^{6}A3	v^{5}A2	vM2	v^{30}A^{8}4	1\15
22	(P8/5, P12/6)	3\15	1\15	^^M2	v^{5}m2	0\15	/m3	/^{6}dd2	1\15	(P8/5, WM7/30)	^^{12}d^{3}2	v^{5}A3	^^{13}d^{4}1	v^{30}A^{9}2	1\15
23	(P8/5, WWP4/6)*	3\15	0\15	^^M2	v^{5}m2	0\15	\4	\$^{6}$m2	0\15	(P8/5, W^{5}m9/30)	^^{12}M2	v^{5}4	^^{13}M2	v^{30}m2	0\15
24	(P8, P11/7)*	15\15	3\15	P8			^^{3}M2	v^{7}m2	0\15	(P8, W^{5}P5/7)					
25	(P8/3, P11/7)	5\15	2\15	vM3	^^{3}d2	1\15	/^{3}M2	\$^{7}$m2	0\15	(P8/3, WWM6/21)	v^{7}AA3	v^{9}A^{3}2	^^m2	^^{21}d^{7}2	1\15
26	(P8/5, P11/7)*	3\15	0\15	^^M2	v^{5}m2	0\15	/^{3}M2	\$^{7}$m2	0\15	(P8/5, m2/35)	^^{14}M2	^^{15}M2	^1	v^{35}m2	0\15
27	(P8, P12/8)*	15\15	3\15	P8			^^{3}M2	v^{8}m2	0\15	(P8, W^{6}P4/8)					
28	(P8/3, P12/8)	5\15	2\15	vM3	^^{3}d2	1\15	/^{3}M2	\$^{8}$m2	0\15	(P8/3, W^{3}m3/24)	^^{8}dd3	^^{9}dd2	vM2	v^{24}A^{7}2	1\15
29	(P8/5, P12/8)*	3\15	0\15	^^M2	v^{5}m2	0\15	/^{3}M2	\$^{8}$m2	0\15	(P8/5, m2/40)	^^{16}M2	^^{15}M2	^1	v^{40}m2	0\15
30	(P8, P5/9)	15\15	1\15	P8			^^{4}d1	v^{9}AA2	1\15	(P8, W^{8}P4/9)					
31	(P8, WWP4/9)	15\15	4\15	P8			^m3	^^{9}d^{3}2	1\15	(P8, W^{6}P5/9)					
32	(P8, W^{3}P5/9)*	15\15	6\15	P8			vv4	v^{9}m2	0\15	(P8, W^{5}P4/9)					
33	(P8/3, P5/9)	5\15	1\15	vM3	^^{3}d2	1\15	/^{4}d1	\$^{9}$AA2	1\15	(P8/3, WWP4/9)					
34	(P8/5, P5/9)	3\15	1\15	^^M2	v^{5}m2	0\15	/^{4}d1	\$^{9}$AA2	1\15	(P8/5, W^{5}m9/45)	v^{18}A^{6}2	v^{20}A^{7}1	^^m2	^^{45}d^{15}2	1\15
35	(P8/5, WWP4/9)	3\15	1\15	^^M2	v^{5}m2	0\15	/m3	/^{9}d^{3}2	1\15	(P8/5, WWm9/45)	^^{18}d^{5}2	v^{5}A3	^^{22}d^{7}1	v^{45}A^{14}2	1\15
36	(P8/5, W^{3}P5/9)*	3\15	0\15	^^M2	v^{5}m2	0\15	\\4	\$^{9}$m2	0\15	(P8/5, W^{8}M7/45)	^^{18}M2	v^{10}4	^^{17}M2	v^{45}m2	0\15

Pergens supported by 19edo. Edos that are a prime number support only 1 pergen per block.

alt-pergenLister: find all rank-2 pergens (Tall Kite Software)

Edit...

first pergen displayed 1.0
maximum fraction 50.0
EDO to be compatible with (0 = none) 19.0

	PERGEN	Per/Gen cents		Per	Enhar1	^1 cents	Gen	Enhar2	^1 cents	Unreduced Pergen	Per	Gen	altGen	Enhar3	^1 cents
1	(P8, P5)	19\19	11\19	P8			P5			(P8, P4)					
2	(P8, P4/2)	19\19	4\19	P8			^M2	vvm2	1\19	(P8, P12/2)					
3	(P8, P11/3)	19\19	9\19	P8			^4	v^{3}M2	1\19	(P8, P12/3)					
4	(P8, P4/4)	19\19	2\19	P8			^m2	^^{4}dd2	0\19	(P8, W^{3}P5/4)					
5	(P8, P12/5)	19\19	6\19	P8			vM3	^^{5}dd2	0\19	(P8, W^{3}P4/5)					
6	(P8, P12/6)	19\19	5\19	P8			^m3	^^{6}dd2	0\19	(P8, W^{4}P4/6)					
7	(P8, WWP5/7)	19\19	7\19	P8			^^{3}d4	^^{7}dd2	0\19	(P8, W^{4}P4/7)					
8	(P8, P4/8)	19\19	1\19	P8			v^{3}A1	^^{8}dd2	0\19	(P8, W^{7}P5/8)					
9	(P8, P11/9)	19\19	3\19	P8			vM2	^^{9}dd2	0\19	(P8, W^{7}P5/9)					
10	(P8, P12/10)	19\19	3\19	P8			vM2	^^{10}dd2	0\19	(P8, W^{8}P4/10)					
11	(P8, P5/11)	19\19	1\19	P8			v^{4}A1	^^{11}dd2	0\19	(P8, W^{10}P4/11)					
12	(P8, W^{4}P4/12)	19\19	7\19	P8			^^{5}d4	^^{12}dd2	0\19	(P8, W^{7}P5/12)					
13	(P8, W^{3}P4/13)	19\19	5\19	P8			^^m3	^^{13}dd2	0\19	(P8, W^{9}P5/13)					
14	(P8, W^{4}P4/14)	19\19	6\19	P8			v^{3}M3	^^{14}dd2	0\19	(P8, W^{9}P5/14)					
15	(P8, P12/15)	19\19	2\19	P8			^^{4}m2	^^{15}dd2	0\19	(P8, W^{13}P4/15)					
16	(P8, W^{7}P5/16)	19\19	9\19	P8			v^{5}A4	^^{16}dd2	0\19	(P8, W^{8}P4/16)					
17	(P8, W^{3}P5/17)	19\19	4\19	P8			v^{8}A2	^^{17}dd2	0\19	(P8, W^{13}P4/17)					
18	(P8, W^{7}P5/18)	19\19	8\19	P8			^4	^^{18}dd2	0\19	(P8, W^{10}P4/18)					
19	(P8, W^{8}P4/20)	19\19	8\19	P8			^4	^^{20}dd2	0\19	(P8, W^{11}P5/20)					
20	(P8, W^{4}P4/21)	19\19	4\19	P8			v^{10}A2	^^{21}dd2	0\19	(P8, W^{16}P5/21)					
21	(P8, W^{10}P4/22)	19\19	9\19	P8			v^{7}A4	^^{22}dd2	0\19	(P8, W^{11}P5/22)					
22	(P8, WWP4/23)	19\19	2\19	P8			^^{6}m2	^^{23}dd2	0\19	(P8, W^{20}P5/23)					
23	(P8, W^{7}P5/24)	19\19	6\19	P8			v^{5}M3	^^{24}dd2	0\19	(P8, W^{16}P4/24)					

Listing all valid pergens is not a trivial task, like listing all valid edos or all valid MOS scales. Not all combinations of octave fractions and multigen fractions make a valid pergen. The search for rank-2 pergens can be done by looping through all possible square mappings [(x, y), (0, z)], and using the formula (P8/x, (i·z - y, x) / xz). While x is always positive and z is always nonzero, y can take on any value. For any x and z, y can be constrained to produce a reasonable cents value for 3/1. Let T be the tempered twefth 3/1. The mapping says T = y·P + z·G = y·P8/x + z·G. Thus y = x·(T/P8 - z·G/P8). We adopt the convention that G is less than half an octave. We constrain T so that the 5th is between 600¢ and 800¢, which certainly includes anything that sounds like a 5th. Thus T is between 3/2 and 5/3 of an octave. We assume that if the octave is stretched, the ranges of T and G will be stretched along with it. The outer ranges of y can now be computed, using the floor function to round down to the nearest integer, and the ceiling function to round up:

If z > 0, then y is at least ceiling (x·(3/2 - z/2)) and at most floor (x·5/3)
If z < 0, then y is at least ceiling (x·3/2) and at most floor (x·(5/3 - z/2))

Next we loop through all combinations of x and z in such a way that larger values of x and z come last:

```
i = 1; loop (maxFraction,
    j = 1; loop (i - 1,
        makeMapping (i, j); makeMapping (i, -j);
        makeMapping (j, i); makeMapping (j, -i);
        j += 1;
    );
    makeMapping (i, i); makeMapping (i, -i);
    i += 1;
);
```

The makeMapping function uses the two parameters as x and z, and loops through all valid values of y. Every value of i from -x to x is tested, and the one that minimizes the multigen's splitting fraction and cents is chosen. This combination of x, y, z and i makes a valid pergen. If the pergen is of the form (P8/m, P4), it's converted to (P8/m, P5). This pergen is added to the list, unless it's a duplicate. The pergens are almost but not quite in the proper order, they need to be sorted. Experimenting with allowing y and i to range further does not produce any additional pergens.

Various proofs (unfinished)

Although not yet rigorously proven, the two false-double tests have been empirically verified by alt-pergenLister.

The interval P8/2 has a "ratio" of the square root of 2, which equals $2^{1/2}$, and its monzo can be written with fractions as (1/2, 0). In general, the pergen (P8/m, (a,b)/n) implies P = (1/m, 0) and G = (a/n, b/n). These equations make the **pergen matrix** [(1/m 0) (a/n b/n)], which is P and G in terms of P8 and P12. Its inverse is [(m 0) (-am/b n/b)], which is P8 and P12 in terms of P and G, i.e. the square mapping.

Because we started with a valid pergen, the square mapping must be an integer matrix. Since n/b is an integer, n must be a multiple of |b|. From this it follows that a and b must be coprime, otherwise a, b, and n could all be reduced by GCD (a,b), and the multigen could be simplified. Since GCD (a, b) = 1 and -am/b is an integer, it follows that m must be a multiple of |b| as well.

Thus GCD (m,n) = |b| · GCD (m/|b|, n/|b|) = |b| · r. If r = 1, then GCD (m, n) = |b|, and vice versa, which is the proposed test for a false double.

If m = |b|, is the pergen explicitly false? Does splitting (a,b) into n generators also split P8 into m periods?
(a+b)·P8 = (a+b,0) = (a,b) - (-b,b) = M - b·P5
Because n is a multiple of b, n/b is an integer
M/b = (n/b)·M/n = (n/b)·G
(a+b)·P8 = b·(M/b - P5) = b·((n/b)·G - P5)
Let c and d be the bezout pair of a+b and b, with c·(a+b) + d·b = 1
Since the pergen is a double-split, m > 1, therefore |b| > 1, therefore c ≠ 0
c·(a+b)·P8 = c·b·((n/b)·G - P5)
(1 - d·b)·P8 = c·b·((n/b)·G - P5)
P8 = d·b·P8 + c·b·((n/b)·G - P5) = b · (d·P8 + c·(n/b)·G - c·P5)
P8/m = P8/|b| = sign (b) · (d·P8 - c·P5 + c·(n/b)·G)
Therefore P8 is split into m periods
Therefore if m = |b|, the pergen is explicitly false

Assume the pergen is a false double, and there's a comma C that splits both P8 and (a,b) appropriately. Can we prove r = 1? Let Q = the higher prime that C uses. Express P, G and C as monzos of the prime subgroup 2.3.Q, by expanding the 2x2 pergen matrix to a 3x3 matrix A:

P = (1/m, 0, 0)
G = (a/n, b/n, 0)
C = (u, v, w)

Here u, v and w are integers. If GCD (u, v, w) > 1, simplify C so that it = 1. The inverse of A expresses 2, 3 and Q in terms of P, G and C. If C is tempered out, the C column can be discarded, making the usual 3x2 period-generator mapping. However, if C is not tempered out, the inverse of A is a 3x3 period-generator-comma mapping, which is simply a change of basis. For example, 5-limit JI can be generated by 2/1, 3/2 and 81/80. Here is the inverse of A:

2 = 2/1 = P8 = (m, 0, 0) · (P, G, C)
3 = 3/1 = P12 = (-am/b, n/b, 0) · (P, G, C)
Q = Q/1 = ((av-bu)m/wb, -vn/wb, 1/w) · (P, G, C)

Fractions are allowed in the first two rows of A but not the 3rd row. Fractions are allowed in the last column of A-inverse, but not the first two columns. Every pergen except the unsplit one requires |w| > 1, so the last column almost always has a fraction. To avoid fractions in the first two columns, A must be unimodular ***[I think, not sure]***, and we have wb/mn = ±1, and w = ±mn/b. Substituting for w, we have:

2 = 2/1 = P8 = (m, 0, 0) · (P, G, C)
3 = 3/1 = P12 = (-am/b, n/b, 0) · (P, G, C)
Q = Q/1 = (±(av-bu)/n, ±(-v)/m, ±b/mn) · (P, G, C)

For v/m to be an integer, v must equal i·m for some integer i. Likewise, av-bu must equal j·n for some integer j. Thus bu = av - jn = iam - jn. Let p = m/rb and q = n/rb, where p and q are coprime integers, nonzero but possibly negative. Then m = prb and n = qrb. Substituting, we get bu = iaprb - jqrb, and u = r(iap - jq). Furthermore, v = im = iprb and w = ±mn/b = ±pqrrb. Thus u, v and w are all divisible by r. If r > 1, this contradicts the requirement that GCD (u, v, w) = 1, therefore r must be 1, and GCD (m, n) = |b|, and all false doubles pass the false-double test.

Assuming r = 1, can we prove the existence of C = (u, v, w) for some prime Q?

Assume r = 1 and GCD (m, n) = 1. Let G be the generator, with a 2.3.Q monzo of the form (x, y, ±1) for some unspecified higher prime Q. x and y are chosen so that the cents of (a,b) is about n times the cents of G. If the pergen is explicitly false, with m = |b|, the 2.3.Q comma C can be found from the 2nd half of the pergen: (a,b) + C = n·G, and C = (n·x - a, n·y - b, ±n). Obviously C splits (a,b) into n parts. Does it split P8 into m parts? Let q = nr/b as before, but with r = 1, it simplifies to q = n/b.
a·P8 + C = (n·x, n·y - b, ±n) = (qbx, qby - b, ±qb) = b·(q·x, q·y - 1, ±q)
Thus a·P8 splits into b parts, and since m = |b|, a·P8 splits into m parts. Proceed as before with a bezout pair to find the monzo for P8/m.

Next, assume the pergen isn't explicitly false. The unreduced form is (P8/m, (n - am, -bm) / mn). Substituting in m = pb and n = qb, with p and q coprime, we get (P8/m, (q - ap, -pb) / pqb).

Assume the pergen is a true double, and r > 1. Then P8 = m·P = prb·P = r·(pb·P), and P8 splits into (at least) r parts. Furthermore, P12 = (-am/b)P + (n/b)G = -apr·P + qr·G = r·(-ap·P + q·G), and P12 also splits into at least r parts. Thus every 3-limit interval splits into at least r parts.

Given a pergen (P8/m, (a,b)/n), how many parts is an arbitrary interval (a',b') split into?
(a',b') = (a'·b, b'·b) / b = (a'·b - a·b', 0) / b + (a·b', b'·b) / b = (a'·b - a·b')·P8 / b + b'·(a,b) / b = (a'·b - a·b')·(m/b)·P + b'·(n/b)·G
Therefore (a',b') is split into GCD (a'·b - a·b')·(m/b), b'·(n/b)) parts.
If m = 1, then b = ±1, and we have GCD (a' ± a·b', b'·n)
If n = 1, then a = -1 and b = 1, and we have GCD (a'·m + b'·m, b') = GCD (a'·m, b')
If m = 1 and n = 1, we have GCD (a', b') = the naturally occurring split.
If m = n (nth-everything), we have n · GCD (a', b')

The multigen and the arbitrary interval can be expressed as gedras:
(a,b) = [k,s] = (-11k+19s, 7k-12s)
(a',b') = [k',s'] = (-11k'+19s', 7k'-12s')
a'·b - a·b' works out to be k·s' - k'·s, and we have GCD ((k·s' - k'·s)·m/b, b'·n/b)
If s is a multiple of n (happens when E is an A1) and s' is a multiple of n, let s = x·n and s' = y·n
GCD ((k·y·n - k'·x·n)·m/b, b'·n/b) = (n/b) · GCD (x·m·(y·k - k'), b')
Thus every such interval is split, e.g. the half-5th pergen splits every 3rd, 5th, 7th, 9th and 11th, including aug, dim, major and minor ones. This is an easy way to determine if an interval is split or not by the pergen.

To prove: if r = 1, it's a false double, and (a,b)/n splits P8 into m parts
if r > 1, it's a true double
a·P8 = (a,0) = (a,b) - (0,b) = M - b·P12
M = n·G = qrb·G
a·P8 = qrb·G - b·P12 = b·(qr·G - P12)
Let c and d be the bezout pair of a and b, with c·a + d·b = 1
If |b| = 1, let c = 1 and d = ±a, to avoid c = 0
ca·P8 = cb·(qr·G - P12)
(1 - d·b)·P8 = c·b·(qr·G - P12)
P8 = d·b·P8 + c·b·(qr·G - P12) = b · (d·P8 + cqr·G - c·P12) = b · (d,-c) + bcqr·G

Miscellaneous Notes

Glossary (to find the definition, use control-F to search for the bolded occurence of the word on this page)
pergen
split
multigen
ups and downs (the ^ and v symbols)
higher prime (any prime > 3)
color depth
dependent/independent
square mapping
highs and lows (the / and \ symbols)
enharmonic
genchain
perchain
wide/widen (increased by an octave)
single-split, double-split
single-pair, double-pair (number of new accidentals in the notation)
true double, false double
explicitly false
unreduced
alternate vs. equivalent (generator or period)
mapping comma
keyspan
stepspan
gedra
edomapping

Combining pergens

Tempering out 250/243 creates third-4th, and 49/48 creates half-4th, and tempering out both commas creates sixth-4th. Therefore (P8, P4/3) + (P8, P4/2) = (P8, P4/6). If adding a comma to a temperament doesn't change the pergen, it's a strong extension, otherwise it's a weak extension.

General rules for combining pergens:

- (P8/m, M/n) + (P8, P5) = (P8/m, M/n)
- (P8/m, P5) + (P8, M/n) = (P8/m, M/n)
- (P8/m, P5) + (P8/m', P5) = (P8/m", P5), where m" = LCM (m,m')
- (P8, M/n) + (P8, M/n') = (P8, M/n"), where n" = LCM (n,n')

However, (P8/2, M2/4) + (P8, P4/2) = (P8/4, P4/2), so the sum isn't always obvious.

If a false double pergen can be broken down into two simpler ones, that may help with finding a double-pair notation with an enharmonic of a 2nd or less. For example, sixth-4th's single pair notation has an E of a 4th. But since sixth-4th is half-4th plus third-4th, and those two have a good E, sixth-4th can be notated with one pair from half-4th and another from third-4th.

Expanding gedras to 5-limit

Gedras can be expanded to 5-limit or higher by including another keyspan that is compatible with 7 and 12, such as 9 or 16. But a more useful approach is for the third number to be the comma 81/80. Thus 5/4 would be a M3 minus a comma, [4, 2, -1]. We can use 64/63 to expand to the 7-limit. For (a,b,c,d) we get [k,s,g,r]:

k = 12a + 19b + 28c + 34d
s = 7a + 11b + 14c + 20d
g = -c
r = -d

a = -11k + 19s - 4g + 6r
b = 7k - 12s + 4g - 2r
c = -g
d = -r

Height of a pergen

The LCM of the pergen's two splitting fractions could be called the **height** of the pergen. For example, (P8, P5) has height 1, and (P8/2, M2/4) has height 4. In single-pair notation, the enharmonic interval's number of ups or downs is equal to the height. The minimum number of ups or downs needed to notate the temperament is half the height, rounded down. If the height is 4 or 5, double-ups and double-downs will be needed.

Credits

Pergens were discovered by Kite Giedraitis in 2017, and developed with the help of Praveen Venkataramana.

(http://www.wikispaces.com/user/view/TallKite)

Help wanted
TallKite (http://www.wikispaces.com/user/view/TallKite) Mar 29, 2018

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